Optimal Downlink Scheduling Policies for Slotted Wireless Time-Varying Channels

V. Tsibonis, L. Georgiadis

Abstract

We consider the problem of scheduling transmissions from the base station to a number of mobile users sharing the same wireless, slotted, time-varying multirate channel. As a performance measure we consider the throughput assigned to each user and with each user we associate a general reward function that reflects the level of satisfaction for a given throughput allocation. By exploiting the variations in the channel conditions and under reasonable statistical assumptions, we propose a policy that maximizes the long-term reward of the system. Specific choices of the reward functions lead to several fairness criteria.

Keywords

Wireless Scheduling, Time-Varying Channels, Fairness, Stochastic Approximation.

I. Introduction

An important characteristic of wireless channels is that the transmission rate perceived by each user is time-varying. These variations in the channel conditions are due to the different and changing locations of the users and to the fast fading of the transmitted signals. In addition the wireless bandwidth is a scarce resource that needs to be allocated carefully. In such a dynamically changing and harsh environment we seek scheduling policies for achieving general fairness criteria by exploiting the variations in the channel conditions.

The problem of scheduling a time-varying wireless channel has been addressed in the past in several different contexts and for a variety of objectives. Stable scheduling in wireless systems without

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time-varying channels was first addressed in [23]. Stable scheduling policies for time-varying wireless channels restricted to "On-Off" channel state processes have been studied in [24]. Stable scheduling policies for multirate wireless channels have been studied in [21], [18], [22], [13], [1] and [2]. The problem of scheduling transmissions opportunistically in order optimize the wireless resource utilization, under the assumption that all queues operate in saturation, is dealt in [8] and [14]. A paper that bridges the gap between these two approaches (i.e., stable wireless system and saturated wireless system) is [26]. In [26], a scheduling policy that maximizes the weighted sum of channel throughputs under any arrival rates is introduced for the case of "On-Off" channels, by adopting a burstiness-constrained model for the channel availability process.

Another important issue besides efficiency is fairness. Fair allocation of network resources for the wireline case has attracted much attention in the past years and several definitions of fairness have been proposed. One common definition is *max-min* fairness as defined in [5]. An alternative definition has been introduced by Kelly, [11], [12] and is termed *proportional fairness*. In [11] and [12], it is shown that proportional fairness is a special case of utility optimization. In [9] and [11], it is shown that max-min fairness can be obtained as a limiting case of a utility optimization.

The issue of fairness has also been addressed for the wireless case. For "On-Off" channel models, algorithms that achieve fairness in terms of providing bandwidth allocations in proportion of prespecified weights are given in [17], [27], [15], [16], [19]. In essence these algorithms try to approximate the GPS policy [20] for the wireless case, where the existence of location-dependent channel errors prohibits the direct extension of packet fair queueing algorithms from the wireline to the wireless case. For multirate channels proportional fairness has been introduced in [25] and [10]. In [8], opportunistic scheduling is employed so as to maximize the minimum long-term average user throughput in terms of prespecified target throughputs for the multirate channel case. Finally for multirate channels, a scheduling algorithm for the maximization of the average system performance is presented in [14]. In this work fairness is introduced in terms of the time fractions users can access the channel.

The main contribution of this paper is the development of a scheduling policy (actually a family of policies parameterized by the choice of the reward functions) for the *multiuser multirate time-variable channel scheduling problem,* that aims to treat the issue of fairness in a unified way. The system considered is that of a base station transmitting data to users over the same wireless time-slotted channel. The channel quality perceived by each user is time-varying. Hence, data can be transmitted to each user at varying rates. From a user's perspective, the performance measure is its throughput. The system's objective is to allocate the channel throughput fairly to the users. We consider a unified framework for addressing the issue of fairness, specifically utility optimization, and provide an adaptive policy that is optimal with respect to this optimization criterion. An important feature of this policy is that it does not depend on channel statistics. The analysis is based on novel application of stochastic approximation techniques developed in [7]. We believe that this approach is applicable to the study of other wireless systems as well.

The rest of the paper is organized as follows. In Section II the system model is presented. Specifically, the assumptions regarding the channel state process are given. In Section III we provide the problem formulation and the definition of the proposed scheduling policy. The optimality proof of the proposed scheduling policy and its applications on the fair allocation of the wireless channel are given in Section IV. In Section V, we present simulation results that are directed towards assessing the transient behavior, and the ability of the proposed policy to adapt to sudden changes in the channel statistics. Furthermore, the performance of the proposed policy is examined in the presence of channel errors. Finally in Section VI we give the conclusions and suggestions for further work.

II. System model

We consider a system consisting of N mobile data users and one single base station (server). The mobile users are connected with the base station through the same wireless channel. With each user there is an associated queue holding packets that are to be transmitted to the user from the base station (downlink channel). The base station operates in discrete time, i.e., it transmits to the users in time-slots of equal duration. Slot $t \geq 1$ refers to the time interval $(t-1, t]$. At any given time-slot the base station transmits to only one user.

The channel conditions for the various users are time-varying and thus users experience time-varying channel quality. There is a finite set of channel states $\mathcal{J}$ and the channel state is constant within each slot. Associated with each state $j \in \mathcal{J}$, there is a vector of feasible data rates $(R_{1j}, ..., R_{Nj})$, where $R_{ij},\ i \in \mathcal{N},\ j \in \mathcal{J}$ denotes the rate at which transmission to user i takes place in a given slot if the channel is in state j during that slot. Let $R_i\,(t)$ denote the feasible rate for user i at slot t, and $s\,(t)$ the (random) channel state at slot t. Hence, $R_i(t) = R_{is(t)}$.

We assume that the variables $s(t),\ t = 1, 2, ...,$ are independent and identically distributed (i.i.d), with $\Pr(s(t) = j) = \pi_j,\ j \in \mathcal{J}$. This assumption can be weakened considerably (see Section IV-A). We use it in the main derivations in order to reveal the essential aspects of the problem and to avoid several purely technical complications that the generalization entails. Finally, we assume that all queues are saturated, i.e., they always have packets available for transmission.

At the base station a scheduler decides at each slot to which user to transmit. The scheduler is aware of the channel state at the beginning of each slot, i.e., it has perfect knowledge of the feasible rate vector $(R_1\,(t+1)\,, ..., R_N\,(t+1))$, at time t. This can be achieved by a coordination between the base station and the users. The base station provides a pilot signal which is measured by all users. These measurements are then fed back to the base station and can be used for the estimation of the feasible rate vector. This is the approach followed in Qualcomm's High Data Rate (HDR) wireless system [3].

III. Problem formulation

Let Π be the set of scheduling policies employed by the scheduler at the base station. For $u \in \Pi$, denote by $D_i^u\,(t)\,,\ i \in \mathcal{N}$, the number of bits that are transmitted from user's i queue up to time t.

The "throughput" of user i under the scheduling policy u up to time t is

$$r_i^u(t) = \frac{D_i^u(t)}{t}, \ i \in \mathcal{N}, \ u \in \Pi.$$

We address the problem of scheduling the transmissions at the base station (downlink scheduling), in such a way that an objective function of the user throughputs is maximized. Specific choices of the objective function lead to scheduling policies that satisfy various fairness criteria and management objectives (see Section IV-A). More specifically, with user i there is an associated reward function $f_i(r)$, denoting the reward obtained up to time t for throughput $r = r_i^u(t)$. We assume that $\{f_i(\cdot)\}_{i \in \mathcal{N}}$ are concave, nondecreasing, twice continuously differentiable functions defined on $[0, \infty)$.

The *total reward up to time* t, is $F^u(t) = \sum_{i \in \mathcal{N}} f_i(r_i^u(t))$. The *total reward* obtained by policy u is defined as $F^u = \limsup_{t \to \infty} \sum_{i \in \mathcal{N}} f_i(r_i^u(t))$. We are taking lim sup since it may not known a priori whether the limit exists under an arbitrary scheduling policy $u \in \Pi$. Our objective is to design a policy that maximizes the total reward, i.e., we are interested in the problem

Problem (P): *Determine a policy* $\pi^* \in \Pi$ *such that* $F^{\pi^*} \geq F^u$, *for all policies* $u \in \Pi$.

Consider the following scheduling policy.

Scheduling policy π

At any slot t *associate with each user* i *an index* $I_i(r_i^\pi(t-1), R_i(t))$ *of the form*

$$I_i(r_i^\pi(t-1), R_i(t)) = R_i(t) f_i'(r_i^\pi(t-1)).$$

At time-slot t *transmit from the user queue with the largest index* $I_i(r_i^\pi(t-1), R_i(t))$. *If there are multiple such user queues, select one arbitrarily.*

Our objective is to show that policy π solves Problem (P). This will be proved in the framework of stochastic approximation. It is worth noting that at any time-slot t, the scheduling decision is based on the feasible rates seen by the users in slot t, as well and on the measured throughputs of the users up to time $t-1$. Thus the scheduling decision is based on the current channel state, as well and on the past scheduling decisions (i.e., the history of the system).

Regarding the computational complexity, at each time slot the policy has to compute the new throughputs, the new indices and to find the user queue with the largest index. The computation of new throughput per user can be done by a simple recursion (see equation (21) in Section V) in $O(1)$ time. The caclulation of each index takes $O(1)$ time as well and the caclulation of the maximum takes $O(N)$ time. Hence the computational complexity per slot is $O(N)$. Moreover, since the policy needs to keep track only of $r_i^{\pi}(t)$ and $R_i(t)$, the space complexity is also $O(N)$.

A. Related deterministic optimization problem

It will be convenient in the sequel to redefine the user throughputs in terms of auxiliary quantities. To this end define, $\Delta_j(\tau)$ a 0-1 variable indicating whether or not $s(\tau)=j$, and $\delta_i^u(\tau)$ a 0-1 variable indicating whether or not at slot τ policy u selects user i for transmission.

The proportion of time up to t that the channel state is j and policy u selects user i for transmission is

$$y_{ij}^u(t) = \frac{\sum_{\tau=1}^{t} \Delta_j(\tau)\,\delta_i^u(\tau)}{t},\ i \in \mathcal{N},\ j \in \mathcal{J}. \tag{1}$$

In the next lemma, part a) relates the user throughputs $r_i^u(t)$ to the quantities $y_{ij}^u(t)$, while part b) provides certain constraints satisfied by $y_{ij}^u(t)$.

Lemma 1: a) The following relations hold.

$$r_i^u(t) = \sum_{j \in \mathcal{J}} R_{ij} y_{ij}^u(t),\ i \in \mathcal{N}.$$

b) The quantities $y_{ij}^u(t)$, satisfy the following constraints.

$$\limsup_{t\to\infty} \sum_{i\in\mathcal{N}} y_{ij}^u(t) \le \pi_j, \quad y_{ij}^u(t) \ge 0,\ i \in \mathcal{N},\ j \in \mathcal{J}.$$

Proof: a) By definition we have that $R_i(t) = \sum_{j\in\mathcal{J}} R_{ij}\Delta_j(t)$. Using this relation and the definitions, we have for $i \in \mathcal{N}$,

$$\begin{aligned} r_i^u(t) &= \frac{D_i^u(t)}{t} = \frac{\sum_{\tau=1}^t \delta_i^u(\tau) R_i(\tau)}{t} = \frac{\sum_{j\in\mathcal{J}} \sum_{\tau=1}^t R_{ij}\Delta_j(\tau)\delta_i^u(\tau)}{t} \\ &= \frac{\sum_{j\in\mathcal{J}} R_{ij} \sum_{\tau=1}^t \Delta_j(\tau)\delta_i^u(\tau)}{t} = \sum_{j\in\mathcal{J}} R_{ij} y_{ij}^u(t). \end{aligned}$$

b) Since bits from at most one user queue are sent at slot t, it holds $\sum_{i\in\mathcal{N}} \delta_i^u(t) \le 1$ for all t. Hence,

$$\sum_{i\in\mathcal{N}} y_{ij}^u(t) = \frac{\sum_{\tau=1}^t \Delta_j(\tau) \sum_{i\in\mathcal{N}} \delta_i^u(\tau)}{t} \le \frac{\sum_{\tau=1}^t \Delta_j(\tau)}{t}. \tag{2}$$

Since the state process $s(t)$, $t = 1, 2, ...$, is i.i.d, the Strong Law of Large Numbers implies that, $\lim_{t\to\infty} \frac{\sum_{\tau=1}^t \Delta_j(\tau)}{t} = \pi_j$,which together with (2) implies that for $j \in \mathcal{J}$, $\limsup_{t\to\infty} \sum_{i\in\mathcal{N}} y_{ij}^u(t) \le \pi_j$. The fact that $y_{ij}^u(t) \ge 0$ follows from the definitions. ■

Consider now the following deterministic optimization problem.

Deterministic Optimization Problem (D).

$$\textit{Maximize} \sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in\mathcal{J}} R_{ij} y_{ij}\right), \ \textit{subject to} \sum_{i\in\mathcal{N}} y_{ij} \le \pi_j,\ y_{ij} \ge 0,\ i \in \mathcal{N},\ j \in \mathcal{J}.$$

Since the space

$$\mathcal{A} = \left\{ y_{ij} : \sum_{i\in\mathcal{N}} y_{ij} \le \pi_j,\ y_{ij} \ge 0,\ i \in \mathcal{N},\ j \in \mathcal{J} \right\}, \tag{3}$$

is compact and the functions $f_i(\cdot)$ are continuous, there exists a solution $\mathbf{y}^*$ to the deterministic optimization problem (see, e.g., [4, page 540]). The next lemma establishes that no policy in Π can have total reward more than the reward of the solution to Problem (D).

Lemma 2: For any policy $u \in \Pi$ it holds

$$\limsup_{t\to\infty} \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in\mathcal{J}} R_{ij} y_{ij}^u(t) \right) \le \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in\mathcal{J}} R_{ij} y_{ij}^* \right)$$

Proof: The proof is similar to the proof of Lemma 2 in [7]. ■

In the next section we will show that policy π satisfies

$$\lim_{t\to\infty} \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in\mathcal{J}} R_{ij} y_{ij}^\pi(t) \right) = \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in\mathcal{J}} R_{ij} y_{ij}^* \right).$$

This, together with Lemma 2 implies that policy π solves Problem (P).

IV. Optimality proof

This section is dedicated to the proof of the following theorem.

Theorem 3: $\lim\limits_{t\to\infty} \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in J} R_{ij} y_{ij}^\pi(t) \right) = \sum_{i\in\mathcal{N}} f_i \left(\sum_{j\in J} R_{ij} y_{ij}^* \right)$.

As mentioned above, this theorem, in combination with Lemma 2 shows that policy π solves Problem (P).

According to Lemma 1 part a), knowledge of $y_{ij}^u(t)$, $i \in \mathcal{N}$, $j \in \mathcal{J}$, implies knowledge of $r_i^u(t)$. Therefore we can work with the variables $y_{ij}^u(t)$ instead of the throughputs $r_i^u(t)$ and the scheduling policy π can be restated as follows

Scheduling policy π

At any slot t associate with each user i an index $I_i(t) = I_i\left(\sum_{j\in J} R_{ij} y_{ij}^\pi(t-1), R_i(t) \right)$ of the form

$$I_i(t) = R_i(t) f_i' \left(\sum_{j\in J} R_{ij} y_{ij}^\pi(t-1) \right).$$

At slot t transmit from the user queue with the largest index $I_i(t)$. If there are multiple such user *queues, select one arbitrarily.*

Since in this section we deal only with policy π, in order to simplify the notation we eliminate π from all related notations, e.g., we write $y_{ij}(t)$ instead of $y_{ij}^{\pi}(t)$.

In order to bring the problem in an appropriate stochastic approximation setting, we provide first a recursion for $(y_{ij}(t) : i \in \mathcal{N},\ j \in \mathcal{J})_{t=1}^{\infty}$. Using the definitions we have,

$$\begin{aligned} y_{ij}(t+1) &= \frac{\sum_{\tau=1}^{t+1} \Delta_j(\tau)\,\delta_i(\tau)}{t+1} = \frac{\Delta_j(t+1)\,\delta_i(t+1) + \sum_{\tau=1}^{t} \Delta_j(\tau)\,\delta_i(\tau)}{t+1} \\ &= \frac{t}{t+1} \sum_{\tau=1}^{t} \frac{\Delta_j(\tau)\,\delta_i(\tau)}{t} + \frac{1}{t+1} \Delta_j(t+1)\,\delta_i(t+1) \\ &= \frac{t}{t+1} y_{ij}(t) + \frac{1}{t+1} \Delta_j(t+1)\,\delta_i(t+1)\,, \text{ or} \end{aligned}$$

$$y_{ij}(t+1) = y_{ij}(t) + \frac{1}{t+1}\left[\Delta_j(t+1)\,\delta_i(t+1) - y_{ij}(t)\right],\ i \in \mathcal{N},\ j \in \mathcal{J}. \tag{4}$$

We will also need an appropriate Lyapunov function for the system under consideration. It is natural to consider the following function

$$V(\mathbf{y}) = \sum_{i \in \mathcal{N}} f_i\left(\sum_{j \in J} R_{ij} y_{ij}^*\right) - \sum_{i \in \mathcal{N}} f_i\left(\sum_{j \in J} R_{ij} y_{ij}\right), \tag{5}$$

where $\mathbf{y}^*$ is the solution to problem (D). However, $V(\mathbf{y})$ is not a Lyapunov function in the strict sense, since it does not satisfy the usual requirement that $V(\mathbf{y}(t)) \geq 0$ for all t. In fact, it is ensured that $V(\mathbf{y}(t)) \geq 0$ only when $\mathbf{y}(t) \in \mathcal{A}$, while $\mathbf{y}(t)$ does not necessarily belong to $\mathcal{A}$ for any given t. In order to remedy this situation, we define the following function $U(\mathbf{y}) = (V(\mathbf{y}))^+$. If we show that $\lim_{t\to\infty} U(\mathbf{y}(t)) = 0$, then Theorem 3 will follow. Indeed, we have by definition $V(\mathbf{y}) \leq U(\mathbf{y})$ and hence

$$\limsup_{t\to\infty} V(\mathbf{y}(t)) \leq \lim_{t\to\infty} U(\mathbf{y}(t)) = 0. \tag{6}$$

On the other hand, from Lemma 2 it follows that

$$\liminf_{t\to\infty} V\left(\mathbf{y}\left(t\right)\right)=\sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in J} R_{ij}y_{ij}^*\right)-\limsup_{t\to\infty}\sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in J} R_{ij}y_{ij}\left(t\right)\right)\geq 0. \tag{7}$$

Inequalities (6) and (7) imply that $\lim_{t\to\infty} V\left(\mathbf{y}\left(t\right)\right)=0$, and hence

$$\lim_{t\to\infty}\sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in J} R_{ij}y_{ij}\left(t\right)\right)=\sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in J} R_{ij}y_{ij}^*\right)-\lim_{t\to\infty} V\left(\mathbf{y}\left(t\right)\right)=\sum_{i\in\mathcal{N}} f_i\left(\sum_{j\in J} R_{ij}y_{ij}^*\right) \tag{8}$$

as claimed by Theorem 3.

In remains to prove that $\lim_{t\to\infty} U(\mathbf{y}\,(t)\,)$ =0. To show this, we use a result from stochastic approximation whose proof can be found in [7, Theorem 2]. We present a simplified version of the result in the next theorem, which is sufficient for our purposes.

Theorem 4: In $\Re^K$ consider a stochastic sequence $\{\mathbf{y}(t)\}_{t=0}^{\infty}$, which satisfies the recursion,

$$\mathbf{y}(t+1)=\mathbf{y}(t)+\frac{1}{t+1}\mathbf{f}(t+1),\quad t\geq 0. \tag{9}$$

Assume that the following conditions hold:

a) There exists a compact set $\mathcal{A}\subset\Re^{\mathcal{K}}$ such that $\{\mathbf{y}(t)\}_{t=0}^{\infty}$ converges to $\mathcal{A}$ a.s., i.e.,

$$\lim_{t\to\infty}\left(\inf\left\{\left|\mathbf{y}(t)-\mathbf{y}\right|:\mathbf{y}\in\mathcal{A}\right\}\right)=0.$$

b) $\mathbf{f}(t)$ is bounded, i.e., $|\mathbf{f}(t)|\leq B$.

c) There exists a twice continuously differentiable function $V:\Re^K\to\Re$ such that

$$\left\langle\nabla V\left(\mathbf{y}(t)\right),E\left[\mathbf{f}(t+1)\left|\mathbf{y}(\tau),\ \tau\leq t\right]\right\rangle<-V\left(\mathbf{y}(t)\right). \tag{10}$$

Then, $\lim_{t\to\infty} (V(\mathbf{y}(t))^{+} = 0$.

There is an obvious correspondence between the recursions that produce the variables $y_{ij}(t)$, i.e., equations (4) and the recursions in Theorem 4. In particular $f_{ij}(t+1) = \Delta_j(t+1)\delta_i(t+1) - y_{ij}(t)$.

The rest of this section is dedicated to verifying conditions a), b), c), for the problem at hand, with $V(\mathbf{y})$ as defined in (5).

Condition a): Define $\mathcal{A}$ as in (3). The fact that the vector $\mathbf{y}(t) = (y_{ij}(t) : i \in \mathcal{N},\ j \in \mathcal{J})$ converges to $\mathcal{A}$ follows from Lemma 1 part b).

Condition b): By definition it holds $0 \leq y_{ij}(t) \leq 1$, $i \in \mathcal{N}$, $j \in \mathcal{J}$. Moreover, $\Delta_j(t+1)\delta_i(t+1)$ can take only two values, either 0 or 1. Hence $|f_{ij}(t+1)| = |\Delta_j(t+1)\delta_i(t+1) - y_{ij}(t)| \leq 1$,which implies condition b).

Condition c): Verifying this condition constitutes the crux of the argument for the optimality of the policy. We first evaluate the expectation vector $E[\mathbf{f}(t+1)\,|\mathbf{y}(\tau),\ \tau \leq t]$. We have

$$
\begin{aligned}
&E[\Delta_j(t+1)\delta_i(t+1) - y_{ij}(t)\,|\mathbf{y}(\tau),\ \tau \leq t] = E[\Delta_j(t+1)\delta_i(t+1)\,|\mathbf{y}(\tau),\ \tau \leq t] - y_{ij}(t) \\
&= \sum_{l\in\mathcal{J}} E[\Delta_j(t+1)\delta_i(t+1)\,|\mathbf{y}(\tau), \tau \leq t,\ s(t+1) = l]\Pr(s(t+1) = l\,|\mathbf{y}(\tau), \tau \leq t) - y_{ij}(t).
\end{aligned}
$$

Since $\mathbf{y}(\tau), \tau \leq t$ depends only on the channel states $s(\tau)$ for $\tau \leq t$ we have by the i.i.d. assumption on the state process $s(t)$, that $\Pr(s(t+1) = l\,|\mathbf{y}(\tau), \tau \leq t) = \pi_l$. By the definition of $\Delta_j(t+1)$ and the policy π, we have

$$
E[\Delta_j(t+1)\delta_i(t+1)\,|\mathbf{y}(\tau), \tau \leq t,\ s(t+1) = l] = \begin{cases} E[\delta_i(t+1)\,|\mathbf{y}(t),\ s(t+1) = l] & \text{if } j = l \\ 0 & j \neq l \end{cases}. \tag{11}
$$

Combining the previous equations, we obtain

$$E\left[f_{ij}(t+1)\left|\mathbf{y}(\tau),\ \tau \le t\right.\right] = E\left[\delta_i\left(t+1\right)\left|\mathbf{y}\left(t\right),\ s\left(t+1\right)=j\right.\right]\pi_j - y_{ij}(t). \tag{12}$$

Note also that from the operation of the policy, we also have

$$E\left[\delta_i\left(t+1\right)\left|\mathbf{y}\left(t\right),\ s\left(t+1\right)=j\right.\right] = \begin{cases} 1 & \text{if } i = \arg\max_{k\in\mathcal{N}}\left\{R_{kj}f_k'\left(\sum_{l\in J} R_{kl}y_{kl}\left(t\right)\right)\right\} \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

We also need to evaluate the term $\nabla V\left(y\left(t\right)\right) = \left(\frac{\partial V(y(t))}{\partial y_{ij}(t)} : i \in \mathcal{N},\ j \in \mathcal{J}\right)$. From (5), we have that

$$\frac{\partial V\left(y\left(t\right)\right)}{\partial y_{ij}\left(t\right)} = -R_{ij}f_i'\left(\sum_{l\in J} R_{il}y_{il}\left(t\right)\right) \tag{14}$$

Combining (12), (13) and (14), we obtain

$$\begin{aligned} &\left\langle \nabla V\left(\mathbf{y}(t)\right), E\left[\mathbf{f}(t+1)\left|\mathbf{y}(\tau),\ \tau\le t\right.\right]\right\rangle = \\ &-\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{N}} R_{ij}f_i'\left(\sum_{l\in\mathcal{J}} R_{il}y_{il}\left(t\right)\right)\left[\pi_j E\left[\delta_i\left(t+1\right)\left|\mathbf{y}\left(t\right),\ s(t+1)=\ j\right.\right] - y_{ij}(t)\right] \\ &= -\sum_{j\in\mathcal{J}} R_{i(j),j}f_{i(j)}'\left(\sum_{l\in\mathcal{J}} R_{i(j),l}y_{i(j),l}\left(t\right)\right)\pi_j + \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{N}} R_{ij}f_i'\left(\sum_{l\in\mathcal{J}} R_{il}y_{il}\left(t\right)\right)y_{ij}(t). \end{aligned} \tag{15}$$

where $i(j) = \underset{k\in\mathcal{N}}{\arg\max}\left\{R_{kj}f_k'\left(\sum_{l\in J} R_{kl}y_{kl}\left(t\right)\right)\right\}$. By definition, the optimal solution to Problem (D), $\mathbf{y}^*$, satisfies $\sum_{i\in\mathcal{N}} y_{ij}^* \le \pi_j,\ j \in \mathcal{J}$. Therefore, taking into account the definition of $i(j)$ and the fact that $f_i'(\cdot)$ is nonnegative (since $f_i(\cdot)$ is nondecreasing), we have

$$\begin{aligned} \sum_{i\in\mathcal{N}} R_{ij}f_i'\left(\sum_{l\in\mathcal{J}} R_{il}y_{il}\left(t\right)\right)y_{ij}^* &\le R_{i(j),j}f_{i(j)}'\left(\sum_{l\in\mathcal{J}} R_{i(j),l}y_{i(j),l}\left(t\right)\right)\sum_{i\in\mathcal{N}} y_{ij}^* \\ &\le R_{i(j),j}f_{i(j)}'\left(\sum_{l\in\mathcal{J}} R_{i(j),l}y_{i(j),l}\left(t\right)\right)\pi_j. \end{aligned} \tag{16}$$

Taking into account (15) and (16) we conclude

$$\left\langle \nabla V\left(\mathbf{y}(t)\right), E\left[\mathbf{f}(t+1)\left|\mathbf{y}(\tau),\ \tau\leq t\right]\right\rangle \leq -\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{N}} R_{ij} f_i' \left(\sum_{l\in\mathcal{J}} R_{il} y_{il}\left(t\right)\right)\left(y_{ij}^* - y_{ij}(t)\right). \quad (17)$$

Using the fact that for a concave function $f(\cdot)$ it holds [4, Proposition B.3], $f(z)-f(x)\leq (z-x)f'(x)$, we have

$$V(\mathbf{y}(t)) = \sum_{i\in\mathcal{N}}\left(f_i\left(\sum_{j\in J} R_{ij} y_{ij}^*\right) - f_i\left(\sum_{j\in J} R_{ij} y_{ij}\right)\right)$$
$$\leq \sum_{i\in\mathcal{N}}\left(\sum_{j\in J} R_{ij} y_{ij}^* - \sum_{j\in J} R_{ij} y_{ij}\right) f_i'\left(\sum_{l\in J} R_{il} y_{il}\right) = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} R_{ij} f_i'\left(\sum_{l\in J} R_{il} y_{il}\right)\left(y_{ij}^* - y_{ij}(t)\right). (18)$$

Finally, from (17) and (18) we conclude that $\left\langle \nabla V\left(\mathbf{y}(t)\right), E\left[\mathbf{f}(t+1)\left|\mathbf{y}(\tau),\ \tau\leq t\right]\right\rangle \leq -V(\mathbf{y}(t))$, which verifies condition c).

A. Generalizations and applications

The condition that the channel state process $s(t)$, $t=1,2...$, is an i.i.d process can be simplified considerably. In fact, it is sufficient to assume that $s(t)$ is a regenerative process. The proof in this case can be based on the full strength of Theorem 2 in [7]. The technical details in this case are getting significantly more complicated, and are similar to those developed in [7]. However, the main points of the argument, specifically the representation of throughputs through the variable $y_{ij}(t)$, the Lyapunov function and the verification of condition c) in Theorem 4 remain the same.

The condition on the reward functions can also be weakened slightly. It is sufficient to assume that $\{f_i\left(\cdot\right)\}_{i\in\mathcal{N}}$ are concave, nondecreasing and continuously differentiable functions on $\Re$, with the property that for $M>0$ and $y,\ h\in\Re^N$ such that $|y|\leq M$, $|h|\leq 1$, there exists a constant $C\geq 0$, depending on M, such that

$$\left|f_i\left(y_i+h_i\right)-f_i\left(y_i\right)-f_i'\left(y_i\right)h_i\right|\leq Ch_i^2,\ i\in\mathcal{N}. \tag{19}$$

In particular, functions with piecewise continuous derivatives can be used.

The choice of the functions $\{f_i(\cdot)\}_{i\in\mathcal{N}}$ depends on the application. For example, if proportional fair sharing is desired, then one can chose $f_i(x)=\ln(x)$ [11], [12].

A minor complication arises here since $\ln(x)$ is not differentiable at 0, where its value becomes infinite. A way out of this technical difficulty, is to use the functions (ε is a sufficiently small constant) $\phi_i(x)=\ln(x)$, if $x\geq\varepsilon$ and $\phi_i(x)=(x/\varepsilon)+(\ln\varepsilon-1)$, if $x\leq\varepsilon$. These functions, satisfy the conditions (19). Moreover, it can be shown that if the region $\mathcal{A}$ contains a point with positive coordinates (i.e. each user has the potential of obtaining nonzero throughput), then for sufficiently small ε, the optimal point obtained using the functions $\phi_i(x)$ coincides with that obtained using $\ln(x)$. The resulting policy π when the functions $\phi_i(x)$ are used, is the following. At any given time-slot t associate with each user i an index $I_i(t)$ of the form $I_i(t)=R_i(t)/r_i(t-1)$, if $r_i(t-1)\geq\varepsilon$ and $I_i(t)=\varepsilon^{-1}$, if $r_i(t-1)<\varepsilon$. At slot t, schedule for transmission the user with the largest index $I_i(t)$. This policy schedules for transmission users whose instantaneous feasible rate is large with respect to their average rate up to the scheduling decision instant. This is in effect the policy proposed in [10] and by Tse in [25].

If it is considered appropriate to apply the max-min fairness criterion [5], then this point (i.e., lexicographic optimal point) can be approximated by using a family of functions, of the form $f_i^{(m)}(x)=c-g(x)^m$, where c is a constant and $g(\cdot)$ is a differentiable, decreasing, convex and positive function. For example $f_i^{(m)}(x)=1-\frac{1}{x^m}$. By using this family of functions, it is shown in [9] that the allocation of throughputs converges to the max-min allocation as m tends to $+\infty$. These functions are not differentiable at 0. A way out of this technical difficulty, is to use modified functions as was done with the logarithmic functions above.

In general, the reward functions may represent user satisfaction levels for achieving a certain throughput. Also, appropriate choice of reward functions can be used to provide priorities. For example, if the network wishes to provide absolute priority to a set of users whenever the throughput of at least one user in this set falls bellow a certain threshold $r_{\min}$, then piecewise linear functions can be used. Specifically, users in this set are assigned a reward function of the form $\overline{f_i}(x) = \overline{c}x$, if $x \leq r_{\min}$ and $\overline{f_i}(x) = \overline{c}r_{\min} + \overline{c}_1(x - r_{\min})$, if $x > r_{\min}$, where $\overline{c} > \overline{c}_1$ (strictly speaking, we must modify $\overline{f_i}(x)$ so that it is differentiable at $r_{\min}$, which can be easily done; we leave it in this form to simplify the discussion). The rest of the users are assigned a reward function $\underline{f_i}(x)$ that has the same form as $\overline{f_i}(x)$, but the slopes now are $\underline{c}$ and $\underline{c}_1$ with $\underline{c} > \underline{c}_1$. If it holds $\overline{c}R_{\min} > \underline{c}R_{\max}$, where $R_{\min}$ ($R_{\max}$) is the smallest (largest) possible feasible rate, then the scheduling policy gives absolute priority to the users with reward functions $\overline{f_i}(x)$ whenever their minimum throughput falls bellow $r_{\min}$. To see this consider a time-slot t such that $r_i(t-1) \leq r_{\min}$ for some user i with reward function $\overline{f_i}(x)$. The index of this user at time-slot t is $R_i(t)\overline{f_i}'(r_i(t-1)) = R_i(t)\overline{c} \geq \overline{c}R_{\min} > \underline{c}R_{\max} \geq \underline{c}R_j(t) > \underline{c}_1 R_j(t)$, for any user j with reward function $\underline{f_j}(x)$. But $\underline{c}R_j(t)$ or $\underline{c}_1 R_j(t)$ is the index of user j at slot t (depending on whether $r_j(t-1) \leq r_{\min}$ or $r_j(t-1) > r_{\min}$). Therefore users with reward functions $\overline{f_i}(x)$ have absolute priority in slot t.

B. Implementation considerations

The proposed policy π schedules the channel at any given slot by considering the observed throughput realizations up to that slot and the observed feasible rate vector in that slot. As seen from equations $r_i^{\pi}(t) = D_i^{\pi}(t)/t$, $i \in \mathcal{N}$, in the throughput realizations used by the policy, the history is equally weighted. This fact implies that policy π is not able to quickly adapt to sudden changes of channel statistics that take place at a large time instant t. This problem can be surpassed by replacing the $1/(t+1)$ factor in (4) by a small constant $\gamma > 0$. In [6], a rigorous treatment of this situation can be found. There, it is shown that for small γ, the proposed policy is able to adapt to the fluctuations in the channel statistics, and yields performance close to the optimal. Hence the new policy that uses γ instead of the factor $1/(t+1)$ trades off convergence to the optimal point for improved adaptability, a situation which is typical in stochastic approximation.

Another standard approach to overcome this problem is to use measurements of throughput observed within a fixed time window [8].

V. Simulation Results

In this section we present simulations that are directed towards assessing the policy's transient behavior, its ability to adapt to sudden changes in the channel statistics and its performance in the presence of channel errors. We examine the policy that uses the direct measurements of channel throughputs as well as the one that uses γ instead of the factor $1/(t+1)$ and compare their relative merits. Furthermore, we compare the scheduling algorithm proposed in [8] with the policy proposed in this paper. Finally, the performance of the proposed policy is examined in the case where errors in packet transmissions occur.

In the following experiments time is slotted and we assume that there is a single base station (i.e., scheduler) which transmits to three users. The queues (located at the base station) holding packets for the three users are assumed continuously backlogged. The rate at which transmission to a user takes place is time-varying. In particular, except from one experiment, the rate for each user is a random variable taking values on a discrete set with uniform distribution. These random variables are assumed independent. In one experiment a continuous channel model is assumed in which the rates for the three users are independent random variables with conditional exponential distribution. The base station is aware of the feasible rates seen by the users in slot-by-slot fashion. By adaptively employing this information the scheduler decides the user to which transmission will take place in any given slot.

In the first experiment the (independent) feasible rates for the three users are governed by a discrete uniform distribution on the set $\{0, 10, 20\}$ for the first user, on the set $\{150, 200, 250\}$ for the second and on the set $\{50, 110, 170\}$ for the third user. In Figure 1, we see the throughput realizations for the three users under policy π for the first 10,000 slots of operation. Policy π, uses the functions $f_i^{(m)}(x) = 1 - \frac{1}{x^m}$, with $m = 15$, which approximate the max-min fair optimal point. We observe that the throughputs converge in roughly 3,000 slots. The throughput vector is $(10.12, 28.34, 27.01)$ at time-slot 10,000.

In the next experiment we examine the convergence of the policy π^γ. Policy π^γ operates as policy π, but the user "throughputs" used for the scheduling decisions are updated through the equations

$$r_i^{\pi^\gamma}(t+1) = r_i^{\pi^\gamma}(t) + \gamma \left[\delta_i^{\pi^\gamma}(t+1) R_i(t+1) - r_i^{\pi^\gamma}(t)\right], \quad i \in \mathcal{N}. \tag{20}$$

The throughput realizations for the policy π^γ with $\gamma = 0.001$ and for the same channel model and functions as in Figure 1, are given in Figure 2. In Figure 2 we plot the actual throughput received by each user, i.e., we plot the functions $r_i(t) = D_i^{\pi^\gamma}(t)/t$, $i \in \mathcal{N}$, and not the "throughputs" used by policy π^γ which are given by (20). We see that the throughputs converge rapidly, but to a slightly "inferior" point, i.e., at slot 10,000 the throughput vector is $(10.12, 28.21, 26.78)$. This is of course expected since as mentioned in the previous section, policy π^γ trades off convergence to the optimal for improved adaptability.

In the third experiment we examine the ability of policy π to adapt to sudden changes in the channel model statistics. Specifically, we consider a system that operates under the channel model of the two previous examples for the first 10,000 slots. At slot 10,000 the channel model changes and the feasible rate for the first user is governed by a discrete uniform distribution on the set $\{250, 300, 350\}$, while the channel statistics for the second and the third user remain the same, i.e., uniform distribution on the set $\{150, 200, 250\}$ and $\{50, 110, 170\}$ respectively. The functions are once again $f_i^{(m)}(x) = 1 - \frac{1}{x^m}$, with $m = 15$. In Figure 3 we see the user throughputs for 20,000 slots of operation. For the first 10,000 slots the graph is identical to Figure 1. After the change in the channel statistics that takes place at slot 10,000, we plot the "pure" throughputs seen by the users after that slot. That is, we set time to zero by replacing t with $t - 10,000$ for $t \geq 10,001$ in the throughput calculations and we do not take into account the already accumulated throughput, i.e., we set the throughput vector equal to zero just after the change in the channel statistics. However the scheduling decisions are unaware of the change in the channel statistics and are based on the equations

$$r_i^{\pi}(t+1) = r_i^{\pi}(t) + \frac{1}{t+1}\left[\delta_i^{\pi}(t+1) R_i(t+1) - r_i^{\pi}(t)\right], \text{ for all } t. \tag{21}$$

After 140,000 slots (i.e., 130,000 slots after the change) policy π converges to a new point where the throughput vector is $(76.13, 72.71, 70.06)$. However, as we see in Figure 3, at slot 20,000 (i.e., 10,000 slots after the change in the channel statistics) the throughput vector is still $(89.31, 68.57, 66.28)$. It is evident that policy π converges very slowly after the change in the channel statistics.

In the next experiment we examine how policy π^{γ} behaves in the presence of statistic changes. For $\gamma = 0.001$ and for the same system and functions as in Figure 3, the throughputs produced by policy π^{γ} are plotted in Figure 4. Once again we do not plot the "throughputs" used by the policy but the actual throughputs seen by the channels. Furthermore for $t \geq 10,001$ we plot the throughputs received after the change by replacing t with $t - 10,000$ and we do not take into account the already accumulated throughput, i.e., we set the throughput vector equal to zero just after the statistical change. For the channel model that describes the system for $t \geq 10,000$ policy π^{γ} converges to the point $(74.65, 72.62, 69.85)$. In figure 4 we see that at slot $20,000$ (i.e., 10,000 slots after the change in the channel statistics) the throughput vector is $(76.25, 72.27, 69.55)$. From Figure 4 we conclude that policy π^{γ} approximates the new optimal point rapidly after the change in the channel statistics.

Next we compare the performance of our policy with respect to the max-min fairness criterion with the policy proposed in [8]. In [8], a policy that maximizes the minimum user throughput is presented. We consider a continuous channel model with three independent users taken from [8]. Specifically, the feasible rate for user i, $i = 1, 2, 3$, is described by a conditional exponential distribution on some interval $[R_{\min}, R_{\max}]$, i.e., $F_i(r) = G_i^{-1}\left[1 - e^{-\gamma_i(r - R_{\min})}\right]$, $r \in [R_{\min}, R_{\max}]$, with $G_i = 1 - e^{-\gamma_i(R_{\max} - R_{\min})}$. We take $R_{\min} = 10$, $R_{\max} = 400$ and $(\gamma_1, \gamma_2, \gamma_3) = (0.02, 0.01, 0.02)$. Thus the feasible rate for the second user is approximately twice as large in distribution as for the first and third user. In Figure 5, we plot the throughputs assigned to the three users as a function of time for the first 10,000 slots when operating under policy π and using the functions $f_i^{(m)}(x) = 1 - \frac{1}{x^m}$, with $m = 20$. Although we have based the analysis for the optimality of policy π on the assumption of a discrete channel model, we see from Figure 5 that the throughputs assigned by policy π converge for the continuous channel described above as well. At slot 10,000 the throughput vector under policy π is $(38, 40, 38.27)$.

The policy proposed in [8], assigns slot t to the user with the maximum $w_i(t)\, R_i(t)$, and it adaptively adjusts the weights $w_i(t)$ so that the minimum throughput is maximized and all the user throughputs are made equal. For the above channel model the optimal weights are $w^* = (0.424, 0.152, 0.424)$, and for these weights the throughput vector assigned by the policy proposed in [8] is $(39, 38.16, 38.6)$. We observe that the performance of the two algorithms is almost identical. This happens because for this channel parameters, at the max-min optimal point (i.e., lexicographic optimal point) the throughputs are equal and therefore the point assigned by the policy proposed in [8] coincides with the max-min optimal point. In general, lexicographic optimization leads to a point of superior performance than the optimization that simply maximizes the minimum user throughput, since it also maximizes the second minimum, third an so on. Therefore, policy π which approximates the lexicographic optimal point, can achieve in general higher performance than the policy proposed in [8].

In the final experiment we examine the ability of the proposed policy to allocate optimally the channel resources in the presence of erroneous transmissions. In particular, we assume that when transmission to a user takes place in a given slot, then with probability ϵ this transmission is corrupted and with probability $(1 - \epsilon)$ this transmission is correctly received by the user. If a transmission is corrupted in a given slot, then it is assumed that all bits transmitted in that slot are lost (i.e., the channel idles in that slot). In this case, the scheduling decisions are based on the recursions (21), but the term $\delta_i^{\pi}(t+1) R_i(t+1)$ is now multiplied by a binary random variable $I(t+1)$, which is equal to 1 if the transmission at slot $t+1$ is correctly received and to 0, otherwise. Furthermore $\Pr(I(t)=0)=\epsilon$, and $\Pr(I(t)=1)=1-\epsilon$. We consider the same channel model and reward functions as in the first experiment and assume an error probability $\epsilon=0.01$. For this selection, we plot in Figure 6 the throughput realizations for the three users over 100,000 slots together with the throughput realizations when all transmissions are error-free (i.e., the throughput realizations shown in Figure 1). It is observed that for $\epsilon=0.01$ the throughput realizations are very close to the ones when the channel is assumed error-free. In Table I, for the same model we give the throughput allocation after 10,000 slots for other values of the probability of error ϵ as well. Note that in this case, the throughput allocation if an error-free channel is assumed is $(10.12,\ 28.34,\ 27.01)$. It is evident that even for a highly unreliable channel (e.g., for $\epsilon=0.1$) the throughput allocation is adequately close to the optimal allocation when a fully reliable channel is assumed.

VI. Conclusions

We presented a policy for downlink scheduling of packets over a wireless time-varying multirate channel. With each user we associated a function that can be interpreted as the reward obtained for a given throughput allocation. Under reasonable assumptions we proved that the policy maximizes the total reward of the system. This framework allows us to deal with the notion of fairness in a unified way for the wireless case. The scheduling policy is relatively easy to implement since it makes scheduling decisions based on the observed measured rate vector in every given slot and on the throughputs seen by the users up to the scheduling decision instant.

In this paper we have considered a system that operates in saturation, i.e., all queues are always full with packets available for transmission. This assumption facilitates the analysis and allows to get a better insight into the problem at hand. However it is important to extend the ideas of this paper to include packet arrival dynamics, i.e., provide scheduling policies that work well under arbitrary packet arrival rates. It is known that in environments with arbitrary arrival rates throughput performance may degrade [24]. On the other hand, it is also known that policies that stabilize the system if the arrival rates are within the stability region, may not perform well when the system is highly congested [26]. In the latter work, the arrival rates were arbitrary, however the optimization objective addressed was maximization of weighted sum of user throughputs and the channel model was on-off type. We believe that the method presented in this work can be combined with the approach in [26] to provide optimal adaptive policies for general utility optimization and for general arrival rates. However, this would require significant extensions and it is a subject of further study.

Another important issue that needs to be addressed, is to incorporate QoS requirements (delay, packet loss etc.) in the performance requirements.

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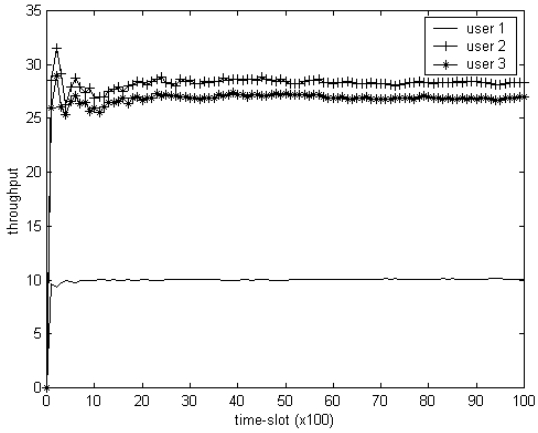

Fig. 1. Throughput for 3 users as a function of time over 10,000 slots (policy π).

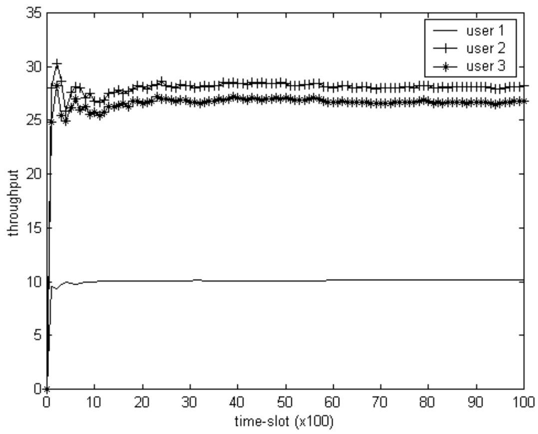

Fig. 2. Throughput for 3 users as a function of time over 10,000 slots (policy π^{γ}).

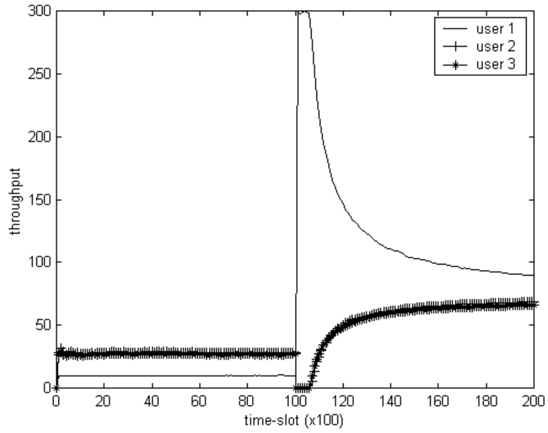

Fig. 3. Throughput adjustment for 3 users in the presence of channel statistics change (policy π).

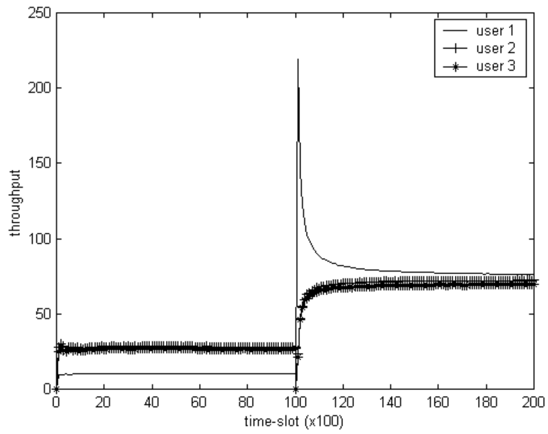

Fig. 4. Throughput adjustment for 3 users in the presence of channel statistics change (policy π^γ).

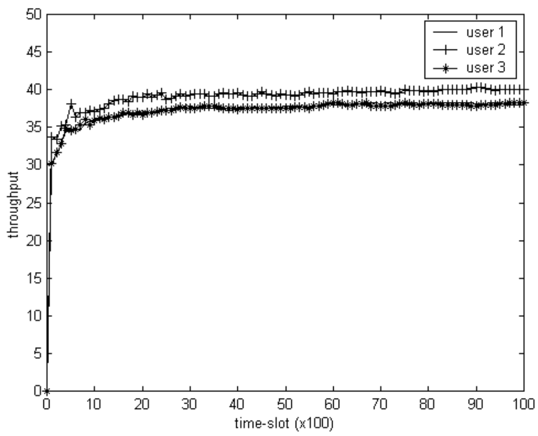

Fig. 5. Throughput for 3 users that perceive a continuous channel over 10,000 slots (policy π).

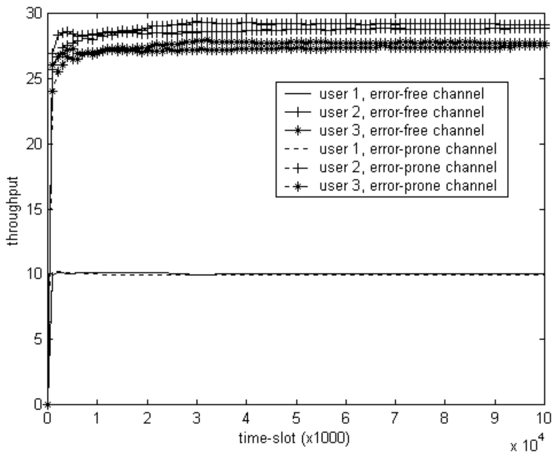

Fig. 6. Throughput for 3 users as a function of time in an error-prone channel (probality of erroneous transmission 0.01).

Prob. of error ϵ	$r_1\,(10,000)$	$r_2\,(10,000)$	$r_3\,(10,000)$
0.01	10.01	28.36	26.87
0.05	9.62	27.37	25.93
0.1	9.12	25.88	24.54

TABLE I

Throughput for 3 users (after 10,000 slots) in an erroneous channel.