

# Load variations impact on optimal DG placement problem concerning energy loss reduction



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## ABSTRACT

The Optimal Distributed Generation Placement problem (ODGP) towards energy loss minimization depends basically on the network's layout and its load composition. Under load variations, different load compositions result, for each one of them, is highly possible to come up with a different optimal solution regarding the optimal siting and sizing of DG units. This paper examines the impact of these variations in order to verify how optimal solution should adapt to any load composition. A Local Particle Swarm Optimization Variant algorithm is proposed as the solution algorithm and numerous load composition snapshots for the IEEE-33 bus system are examined. Moreover, a methodology is proposed in order to highlight the critical nodes that prove to have an essential role to the solution. Finally, the possibility for the determination of a fixed solution with fixed installation nodes and constant power output that could yield near optimal energy loss reduction is examined.

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## 1. Introduction

Incorporation of Distributed Generation (DG) has caused alterations to both the structure and infrastructure of the grid, especially to Distribution Networks (DNs). New challenges regarding the grid-integration of DG have been raised with respect to technical and operational issues that had to be faced through a novel approach [1–4]. On the one hand, these dispersed power units could affect traditional operational aspects of the DNs like the downstream power flow [5], and could cause power quality issues [6]. On the other hand, proper consideration and planning about the siting and sizing of such units could benefit the DN and the upstream grid in many ways. Such benefits are loss reduction [7], voltage improvement [8], reliability improvement [9] and CO<sub>2</sub> emission reduction [10] especially under the installation of Renewable Energy Sources (RESs). This latter approach, namely known as Optimal Distributed Generation Placement (ODGP) problem has attracted a lot of attention during the last two decades in order to offer a guideline towards efficient DG penetration planning.

One of the most common operational issues of DNs related to the ODGP problem refers to the investigation of the optimal siting and sizing of DG units for power loss minimization. This objective is either faced individually [11,12] or constitutes the main one in multi-objective approaches, highly prioritized by a weight factor [13,14]. The optimal solution of the ODGP problem is subject to the simultaneous optimization of the four involved variables [15,16], i.e., the index of the DG hosting nodes, the number and the capacity of the individual DG units, as well as the aggregated DG capacity to be penetrated in the DN. Most of the literature yield biased solutions due to the fact that one or more of the aforementioned variables are predefined. Therefore, some approaches propose a two-stage solution of the problem where the siting part of the problem is firstly solved and then the optimal capacity of the DG units is examined [17,18]. Other studies examine the optimal installation of a small predefined number of DG units [19,20] or investigate how a specific DG capacity with fixed DG units could be optimally allocated [21]. The methods which incorporate either a consecutive solution or a partially predefined input, could produce biased results, different from optimal ones. Moreover, many solution algorithms have been proposed to deal with the problem like analytical methods [22], numerical methods [23] and heuristics [24,25], e.g. genetic algorithms, Particle Swarm Optimization algorithms and differential evolution methods.

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The main disadvantage of most of the existing methodologies relies on the fact that the solution provided refers to a specific snapshot of the DN operation with a fixed load composition. The solution for the ODGP problem depends on the layout of the DN and its load composition. Thus, under load variations, the load composition is accordingly varied, and the solution is expected to alter as well. Therefore, the question raised in such cases is the determination of one representative solution, among all possible, that could be considered optimal regardless the DN load composition.

The impact of DG units on energy losses will depend on the specific characteristics of the network, such as demand profile, topology, as well as the relative location of the generators and whether their output is firm or variable. Incorporating these complexities into an optimization framework for energy loss minimization is a challenge that has only been partially addressed by a few studies [26]. In [27] the analysis regarding load and DG power output variations relies on uniformly distributed loads while these variations refer to a typical daily pattern for both. Moreover, only the optimal siting of DG units is examined, and one DG unit is considered for installation. In [28] the case of one wind power unit under both power output and load demand variations is examined. The analysis yields the optimal node for the wind power unit installation by considering a sequential analysis with only one candidate node for DG installation at a time and concludes that subject to load variations, the optimal location is different when compared to the operational snapshot. In [29] a probabilistic technique is proposed for optimally allocating different types of DG technologies. The technique is based on generating a probabilistic generation-load model. Beta and Rayleigh Probability Density Functions (PDFs) are used for simulating solar irradiance and wind speed uncertainty, respectively, while IEEE-RTS for the load profile. However, the positions of the DGs are predetermined, as the number of DGs as well. Other approaches incorporating load or DG power variations, as in [30], may provide biased solutions since the installation nodes are predetermined. Furthermore, the analysis in [31] concludes that the power analysis of one load snapshot is not necessarily adequate for the overall operation of the DN. Rotaru et al. [32] propose a two-stage method of optimal siting and sizing of DGs. Finally, Shaaban et al. in [33], propose a method to address and evaluate the economic benefits of Renewable DGs when applied to DNs, but the candidate buses are predetermined and the number of DGs for each type is limited and predefined.

In this paper, the ODGP problem subject to load variations is considered as a power oriented problem regarding the net power demand and/or generation of the optimal nodes towards energy loss minimization. The operational snapshot with the average load composition of the DN is considered as the base case scenario and numerous different operational states with altered load compositions are stochastically generated under a uniform distribution with specific deviations. These deviations are properly chosen in order to describe either smooth transitions across the DN load curve, aiming to simulate load alterations concerning daily or weekly load profiles, or more intense load variations that could describe monthly and seasonal load profiles. Next, each stochastically generated scenario is treated as an individual sub-problem, where the same analysis is applied and the ODGP problem is solved with a Local PSO Variant (LPSOV) algorithm. The above scenarios are considered to compile most of the possible operational states of the DN during a year. Thus, the respective solutions for these scenarios constitute the optimal siting and sizing for the DG units that could yield minimum annual energy losses. Although each load composition delivers an individual optimal solution, the analysis is extended to investigate whether a fixed solution for both the installation points and the DG units capacity could yield a near-optimal energy loss reduction. The contribution of this work in power system planning is twofold; firstly, the identification of crit-

ical DN nodes to host DG units, and secondly the determination of the optimal capacities of the latter when considered with fixed power output, towards energy loss minimization.

This paper is organized as follows: in Section 2 the problem formulation along with the LPSOV algorithm and the examined DN are presented. In Section 3, the conceptual justification and mathematical formulation regarding the load variations are analyzed. In Section 4, the results regarding the examined scenarios are presented and discussed while Section 5 is devoted to conclusions.

## 2. Proposed methodology

### 2.1. Problem formulation

The precise computation of energy loss in DNs presumes the availability of real time-series data regarding the actual power flows in all branches. The operators of modern DNs usually keep a measurement log on a 15-min basis, which means that they can assess load variations adequately. Moreover, when a rough estimation is required, the mean load values could be utilized to compute the energy loss for a given period. The accuracy of this estimation depends on the divergence of each node's load variation from the respective considered mean value. The ODGP problem is highly affected by the initial state of the DN in terms of both its topology and load composition. Therefore, under a fixed DN layout, e.g., radial structure with no tie-switch activation, each different snapshot of the DN operational status with altered load composition requires a different solution for the installation nodes and the power output of DG units towards loss minimization. Hence, all different snapshots with altered load composition are expected to yield different solutions for the ODGP problem. A generic objective function which aims at minimizing the energy loss in DNs, taking into account the sum of the sequential snapshots of the network with fixed load compositions, is presented in (1).

$$F_{\text{loss}} = \min \sum_{\Delta t=1}^k \sum_{\substack{i,j=1 \\ i \neq j}}^{\eta_l} g_{i,j} [(V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j))] \quad (1)$$

where

$F_{\text{loss}}$  is the objective function to be minimized,  
 $k$  is the number of different sequential snapshots with fixed load composition that constitute the time period under study,  
 $V_i$  and  $V_j$  are the voltage magnitudes of nodes  $i$  and  $j$ ,  
 $\theta_i$  and  $\theta_j$  are the voltage angles of nodes  $i$  and  $j$ ,  
 $g_{i,j}$  is the conductance between nodes  $i$  and  $j$ ,  
 $\eta_l$  is the total number of branches in the network.

The problem expressed by (1) is subject to the operational constraints:

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (2)$$

$$I_b \leq I_b^{\max} \quad (3)$$

where  $V_i^{\min}$  and  $V_i^{\max}$  are the voltage of node  $i$ , and  $I_b^{\max}$  is the maximum allowed RMS current of branch  $b$ .

The constraints in (2) and (3) along with the conventional constraints regarding the power flow equations are embedded in the objective function as penalty terms [12,34], to reduce the computational burden and facilitate the solution process. A generic penalty function is:

$$P(x) = f(x) + \Omega(x) \quad (4)$$

$$\Omega(x) = \rho \{g^2(x) + [\max(0, h(x))]^2\} \quad (5)$$

where  $P(x)$  is the penalty function,  $f(x)$  is the objective function ( $F_{\text{loss}}$ ),  $\Omega(x)$  is the penalty term,  $\rho$  is the penalty factor,  $g(x)$  is the equality constraints referring to power flow equations and  $h(x)$  is the inequality constraints. Therefore, the Penalty Function (PF) which is used in this work is:

$$PF = \min[F_{\text{loss}} + (\Omega_P + \Omega_Q + \Omega_V + \Omega_L)] \quad (6)$$

with  $\Omega_P$  and  $\Omega_Q$  referring to equality constraints and  $\Omega_V$  and  $\Omega_L$  to inequality constraints respectively [15], and their formulation is as follows:

The equality constraints, i.e. the power flow equations, are presented below:

$$P_{G,i} - P_{D,i} - \sum_{j=1}^{n_b} |V_i||V_j||Y_{i,j}| \cos(\delta_{i,j} - \theta_i + \theta_j) = 0 \quad (7)$$

$$Q_{G,i} - Q_{D,i} + \sum_{j=1}^{n_b} |V_i||V_j||Y_{i,j}| \sin(\delta_{i,j} - \theta_i + \theta_j) = 0 \quad (8)$$

where

$P_{G,i}$  is the real power generation on bus  $i$ ,  
 $Q_{G,i}$  is the reactive power generation on bus  $i$ ,  
 $P_{D,i}$  is the real power demand on bus  $i$ ,  
 $Q_{D,i}$  is the reactive power demand on bus  $i$ ,  
 $n_b$  is the total number of network's buses,  
 $Y_{i,j}$  is the magnitude of bus admittance element  $i, j$ ,  
 $\delta_{i,j}$  is the angle of bus admittance element  $i, j$ .

Thus, the equality penalty terms, i.e.  $\Omega_P$  and  $\Omega_Q$ , are formulated as follows:

$$\Omega_P = \rho_P \sum_{i=1}^{n_b} \{P_{G,i} - P_{D,i} - \sum_{j=1}^{n_b} |V_i||V_j||Y_{i,j}| \cos(\delta_{i,j} - \theta_i + \theta_j)\}^2 \quad (9)$$

$$\Omega_Q = \rho_Q \sum_{i=1}^{n_b} \{Q_{G,i} - Q_{D,i} + \sum_{j=1}^{n_b} |V_i||V_j||Y_{i,j}| \sin(\delta_{i,j} - \theta_i + \theta_j)\}^2 \quad (10)$$

whereas the inequality penalty terms, i.e.  $\Omega_V$  and  $\Omega_L$ :

$$\Omega_V = \rho_V \sum_{i=1}^{n_b} \{\max(0, V_i^{\min} - V_i)\}^2 + \rho_V \sum_{i=1}^{n_b} \{\max(0, V_i - V_i^{\max})\}^2 \quad (11)$$

$$\Omega_L = \rho_L \sum_{b=1}^{n_l} \{\max(0, I_b - I_b^{\max})\}^2 \quad (12)$$

In this work, the stochastic model of the bus load variability refers to the net bus load. That means that it includes the power demand variability as well as any power generation variability which may emerge due to intermittent DGs and RESs.

## 2.2. PSO for ODGP problem

Power loss minimization under ODGP constitutes a complex non-linear mixed integer optimization problem. Since the problem in this paper is examined under an energy loss minimization perspective, the computational burden is multiplied by the number of  $k$  sequential problems that have to be solved. Moreover, in order to ensure a non-biased solution, all problem variables should be simultaneously optimized [15], which comes with an additional computational effort. In this paper, the LPSOV algorithm is used,

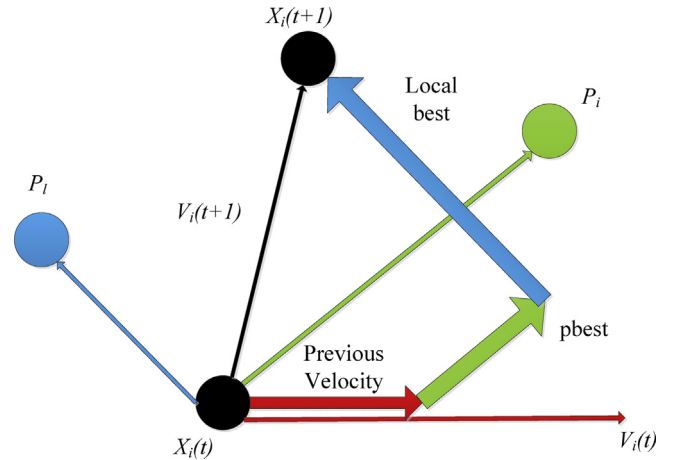


Fig. 1. Vector diagram of a particle's motion.

that has been proved to provide efficient solutions in optimal siting and sizing of DG units [16].

PSO was initially introduced in 1995 by Kennedy and Eberhart [35]. It is a population-based algorithm. Among the various PSO variants, in this study, the basic Local PSO variant is used, or LPSOV. It has been proved that improves the swarm's ability to avoid local optima because it provides a better balance between exploration and exploitation of the solution space, compared to the other basic PSO Variant, i.e. Global PSO. [36].

In general, a swarm of particles is designated to explore the solution space. They move around the solution space based upon their personal experience (personal best), that of other particles (social best), and finally that of their former gained velocity, as shown in Fig. 1, and also in Fig. 4 and further explained in (13)–(15).

The personal best Term represents, evidently, the best personal solution each particle has insofar come across. The Social Best Term represents the information exchange among particles within the swarm regarding the best solution found insofar among them. If this information exchange takes place with all the swarm particles then, the Global PSO Variant is formed and the social best is called global best. If the particles form smaller groups with other particles, called neighborhoods, in order to exchange information within them, then the LPSOV is formed, and the social best is called local best. The neighborhood formulation is described in detail in [34]. In this study, for the neighborhood formulation the ring topology is used, as shown in Fig. 2.

Additionally, regarding the algorithm's performance, in Fig. 3 the average convergence of 1000 trials of LPSOV when implemented on IEEE-33 is demonstrated. It is deduced that LPSOV performs rather well, since it achieves in average 97.73% loss reduction. Furthermore, it takes in average less than 100 iterations to reach below 8.44 kW power losses i.e. almost 96% loss reduction.

$$v_i(t+1) = wv_i(t) + c_1R_1(P_i(t) - X_i(t)) + c_2R_2(P_i(t) + X_i(t)) \quad (13)$$

$$X_i(t+1) = X_i(t) + v_i(t+1) \quad (14)$$

$$w(t) = w_{up} - (w_{up} - w_{low}) \frac{t}{T_{\max}} \quad (15)$$

where

$i = 1, b, \dots, N$  and  $N$  is the number of particles,  
 $X_i(t+1)$  its future position,  
 $v_i(t)$  its current velocity,  
 $v_i(t+1)$  its future velocity,  
 $P_i(t)$  its personal best, pbest,  
 $P_i(t)$  its neighborhood's best, lbest,

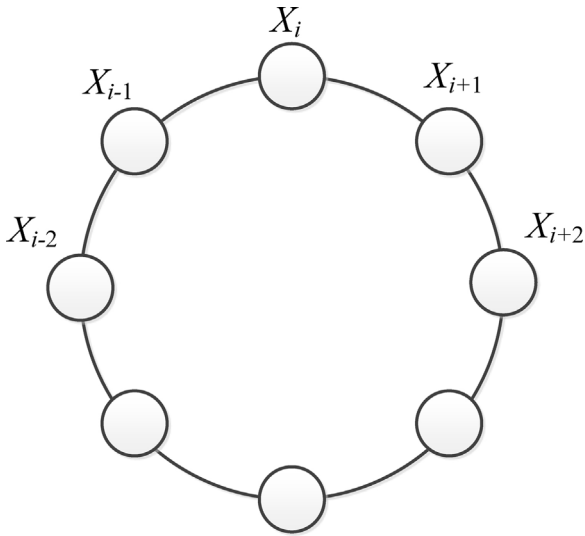


Fig. 2. The ring topology formation and particle's  $X_i$  neighborhood.

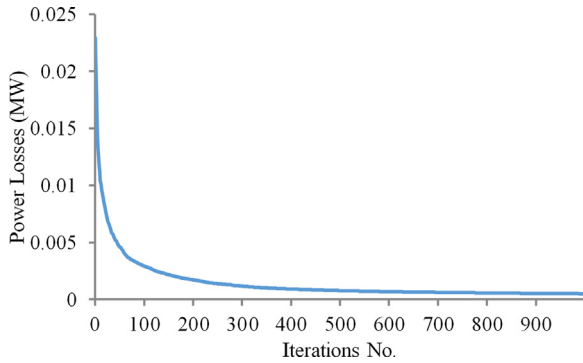


Fig. 3. LPSO's average convergence in IEEE-33 bus systems.

$c_i$  weighting factors, the cognitive and social parameters, respectively,  
 $w$  the inertia weight,  
 $t$  the current iteration number,  
 $w_{up}$  the upper limit of inertia,  
 $w_{low}$  the lower limit of inertia,  
 $T_{max}$  the maximum number of iterations.

The maximum number of iterations along with the convergence tolerance must be both satisfied for the PSO's termination. This approach allows a relatively extensive investigation within the solution space under the requirement for an efficient solution. PSO is, however, a heuristic based algorithm, and thus it cannot guarantee an optimal solution. In order to ensure the less possible impact on the PSO performance, in this work, each solution is selected and delivered by a pool of 50 ones, which is produced by the repeated application of LPSOV in the same problem. This high computational burden for every solution delivery is selected as a trade-off to assure the solution quality, regardless the initial load composition. In Table 1 the values of the algorithm's parameters are summarized. The simulations were conducted on MATLAB environment using MATPOWER [37], in a system of 15 CPU cores @ 3.00 GHz speed and 64 GB RAM.

### 2.3. Test case DN

The proposed methodology is applied to the IEEE-33 bus system [38], a fixed radial DN with 33 nodes. Table 2 contains the initial

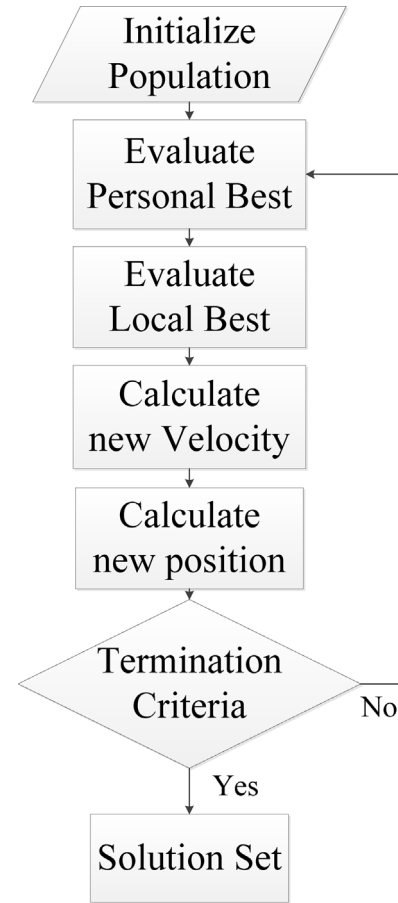


Fig. 4. PSO flowchart.

Table 1  
PSO parameters.

	Variable	Value
$c_i$	: cognitive and social coefficient	2.05
$w_{up}$	: inertia upper limit	0.4
$w_{low}$	: inertia lower limit	0.9
$N$	: number of particles	50
$r$	: neighborhood radius;1;	2
$T_{max}$	: maximum iterations number equal to 1000-	1000
	convergence tolerance	$10^{-7}$

Table 2  
Input data for 33 bus system.

Number of nodes/branches	33/37
Aggregated active/reactive power Production (MW/MVAr)	0/0
Power/voltage base (MVA/kV)	10/12.66
Aggregated active/reactive power Demand (MW/MVAr)	3.72/2.3

data and Fig. 5 presents the initial load composition. This load composition is assumed as the average load composition during the examination of the energy minimization period.

### 3. Load variations

The consideration of the average load composition as the base case scenario relies on the intention to investigate the load variation tolerance that would slightly affect the siting part of the ODGP problem. Proving that the most efficient installation nodes

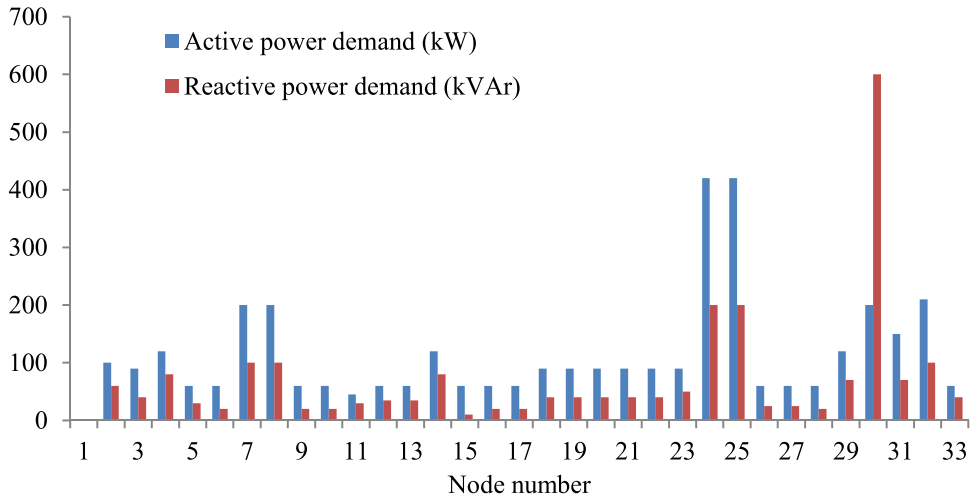


Fig. 5. Initial average load composition of IEEE-33 bus system.

for DGs are not significantly influenced by the network's load composition under not extreme load variations, then a criticality index is assigned to each one in order to prioritize their siting effect for the ODGP problem. The above analysis would be of great importance for the Distribution Network Operator (DNO) since it could provide a guideline for proper strategic planning regarding DG penetration. Within the current liberalized energy market, the DG installation is usually ruled by economic incentives related to private investment plans, or by environmental and geographical constraints and criteria. Nevertheless, the prioritization regarding the critical installation nodes for DG could allow the DNO to promote DG installation at critical nodes to satisfy the specifications defined by the ODGP planning. Although the DNO might be unable to canalize the funds of private investments, proper planning about the prioritization of such investment actions should be performed, and be implemented, whenever possible.

The contribution of the proposed analysis is further expanded to investigate whether a fixed DG capacity at each critical node could be defined to yield the lowest possible energy loss regardless the load variations. Rationally, the DG power output should adapt to the load composition of the DN to cause minimum power losses for each snapshot. The idea here is to examine numerous snapshots with altered load composition under different load variations width to define the optimal fixed DG output for each case. Variation width is assumed to describe shorter or longer time periods of the annual load curve for the DN under study.

In this work, load variations that define different snapshots of the DN load composition, have resulted stochastically using a uniform distribution approach as defined by (7) and (8).

$$S_i^{\text{lower}} = \bar{S}_i \left( 1 - \frac{S_u}{100} \right) \quad (16)$$

$$S_i^{\text{upper}} = \bar{S}_i \left( 1 + \frac{S_u}{100} \right) \quad (17)$$

where  $\bar{S}_i$  is the initial mean load level,  $S_i^{\text{lower}}$  and  $S_i^{\text{upper}}$  are the lower and upper limits of the uniform distribution intervals, respectively, and  $S_u$  is a percentage parameter that defines the length of uniform distribution. The value of  $S_u$  has been set to 20 and 50, i.e. the load of each node for every snapshot  $\Delta t$  is considered to alter randomly within a range of  $\pm 20\%$  and  $\pm 50\%$  from the initial mean value.

The loading condition of the IEEE-24 bus Reliability Test System [39] is studied as a base case, in order to justify the load variations modeling for the present analysis. Its hourly, daily, and weekly peak load factors were used to construct the annual load curve. These peak load factors have been selected in order to capture the loading

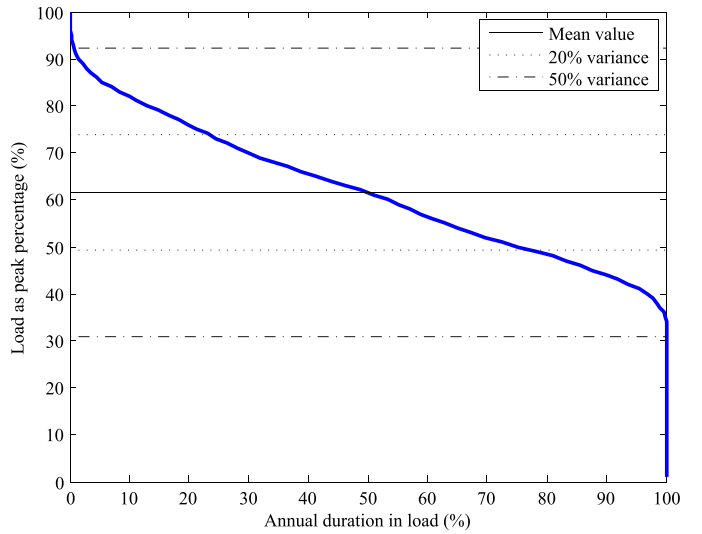


Fig. 6. Load duration curve for IEEE 24 bus system.

conditions that yield the highest annual energy losses and moreover in order to justify the upper limit for the load variations (i.e. 50%) adopted in this analysis. This selected variance could cover a loading composition for the DN that refers to the highest load demands that are expected within a one year time period. Subsequently, the annual load curve is transformed into a cumulative power curve, as shown by the blue line in Fig. 6 to investigate the loading variability. In the same figure, the mean annual power, 61.45% of the annual peak power, along with the 20%, and 50% variance limits are also marked in continuous, dotted, and dashed gray lines, respectively. It is calculated that the 20% and 50% variance cover the 55.08% and 99.40% of the total annual loading levels, respectively. The IEEE-24 bus system is utilized only to justify the load variations magnitude modeled in the analysis. As proved by Fig. 6, the majority of loading conditions of the distribution network during a one year time period could be captured by load variations up to 50% of the average load composition of the network.

It has to be clarified that the variations presented in Fig. 6 refer to the aggregated load demand of the DN. In this work, every snapshot is created individually by the stochastic variation of the net power value of all buses. This approach ensures that each snapshot refers to a different load composition in comparison to all others under the same  $S_u$ . For  $S_u = 20$  the number of snapshots equals 2000 while



**Table 3**  
ODGP solution for initial average load composition.

Node for DG installation	DG active power (kW)	DG reactive power (kVAr)
3	717.72	360.03
6	513.66	244.63
8	541.05	250.50
14	691.46	330.98
25	450.68	723.03
30	420.39	210.41
Initial losses (kW): 211 – loss reduction (%): 97.73		

for  $s_u = 50$  the respective number is 6000. Thus, the ODGP problem has been applied on 8000 different snapshots with different load compositions.

## 4. Results

### 4.1. Best fixed solution

In Table 3 the optimal solution deduced by LPSOV for the ODGP problem concerning the initial average load composition of the IEEE 33 bus system is presented.

The ODGP addressing the snapshots, provides, various optimal solutions. Each solution yields the optimal power loss reduction for the timeslot where the respective snapshot is valid. Alternatively, it could establish the deviation boundaries where a fixed solution is considered as a near optimal one.

Fig. 7a and b refer to the worst and best voltage profiles among the 2000 snapshots for  $s_u = 20$  with the respective improvements after the optimal siting and sizing of DGs. The respective results for  $s_u = 50$  are presented in Fig. 8a and b.

In order to decide the optimal number of the DG hosting nodes for the fixed solution, all 8000 solutions are initially examined in

order to detect the dominant number of installation nodes considered for the optimal solution.

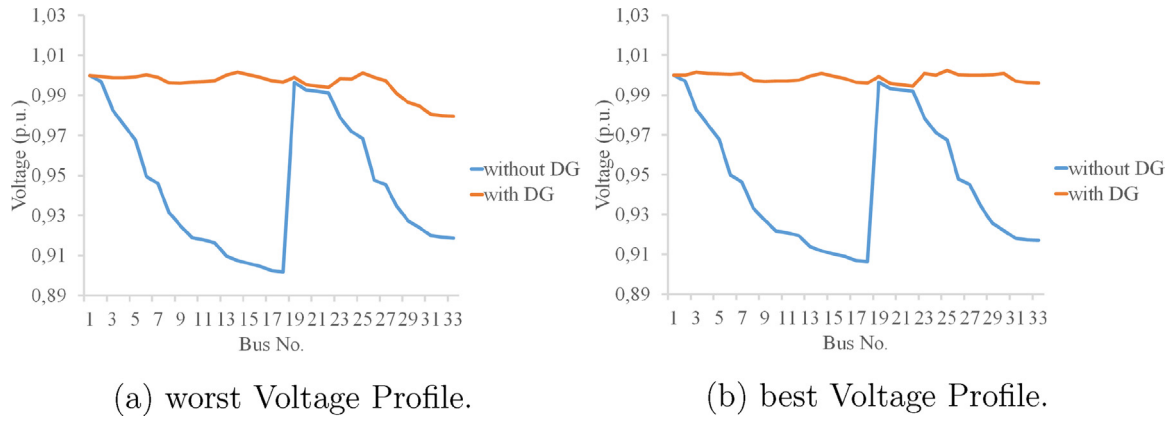
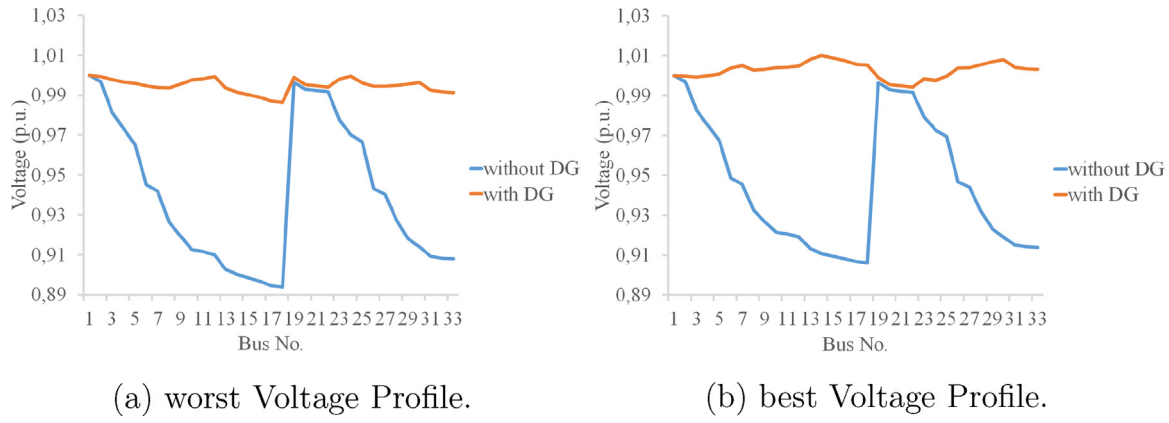
The provided results have been assessed to examine whether any crucial nodes appearing to participate more frequent to the final solution, exist. The algorithm is set free for every single snapshot to reach a solution, unbiased, it reaches one and, most of all, the nodes that tend to appear as part of the solution are the ones that are the most promising/critical nodes of the DN. More importantly, this occurs in every snapshot, thus after examining all the solutions of all the snapshots a pattern develops, where the most promising nodes to host DG units considering all the snapshots, emerge. A weight factor  $wf$  is assigned to each node to quantify its criticality level regarding the ODGP problem. This  $wf$  is formulated based on the information concerning the participation frequency of each node to the solution for all snapshots. More specifically:

1. All nodes are initially placed in descending order based on their participating frequency to the final solution for both  $s_u$  values.
2. For each node, an average value for both active and reactive power output ( $P_{ave}$  and  $Q_{ave}$ ) is computed. These values are the average fixed DG power outputs for the examined time period from the optimal power outputs for the examined snapshots. This value is not probably the optimal one for any snapshot, but it could still be proved to be efficient enough for the aggregated case where all snapshots constitute the examined time period.

In Table 4 the results concerning the prioritization of the nodes for both  $s_u$  values are presented. It is noteworthy that based on the results in Table 4, the eleven most critical nodes for both  $s_u$  values are the same, although their ranking is not equal. Moreover, four out of the first six nodes in Table 4 are also included in the solution for the average load composition under the average load composition of the DN which is presented.

**Table 4**  
Node criticality for the examined snapshots.

$s_u = 20$ (2000 snapshots)				$s_u = 50$ (6000 snapshots)			
Node	$wf$	$P_{ave}$ (kW)	$Q_{ave}$ (kVAr)	Node	$wf$	$P_{ave}$ (kW)	$Q_{ave}$ (kVAr)
30	0.192360	862.742	862.310	30	0.177820	856.193	880.214
14	0.174281	654.118	278.785	14	0.160939	640.686	274.100
7	0.127041	755.487	322.741	25	0.114315	815.422	337.335
25	0.099825	774.886	340.663	7	0.106109	753.127	318.942
24	0.089813	1011.243	447.723	3	0.086850	919.752	286.456
3	0.085828	936.345	330.380	24	0.078912	995.689	431.491
8	0.065902	500.917	210.766	8	0.074357	513.460	206.599
6	0.041407	788.819	295.982	6	0.056873	821.710	305.955
31	0.022162	700.888	199.320	31	0.031987	743.721	372.623
4	0.017010	471.741	159.385	4	0.014536	430.981	199.411
13	0.014580	777.837	308.645	13	0.014469	767.204	294.092
2	0.014386	622.774	251.337	15	0.013666	519.420	222.513
10	0.013025	331.258	143.575	21	0.012326	309.609	128.601
33	0.010400	621.374	204.082	2	0.011221	627.885	257.504
21	0.010206	280.389	129.891	32	0.008976	571.560	195.442
32	0.006124	439.673	202.872	33	0.008541	614.725	173.012
15	0.005152	499.570	212.914	11	0.007235	406.291	131.299
12	0.002624	791.813	175.831	10	0.005761	332.586	152.014
11	0.002527	406.479	170.675	12	0.004421	855.802	265.427
16	0.002430	371.382	168.001	9	0.002747	453.690	97.973
9	0.001166	312.040	145.883	16	0.002613	408.485	138.503
26	0.000778	0	234.762	26	0.001373	611.897	295.741
17	0.000389	295.521	97.024	18	0.001139	147.232	73.873
18	0.000292	0	71.052	17	0.000938	329.882	125.336
20	0.000194	0	117.629	20	0.000737	0	124.057
29	$9.72 \times 10^{-5}$	768.033	847.216	22	0.000703	270.262	122.273
1	0	0	0	29	0.000335	951.151	174.931
5	0	0	0	19	$3.35 \times 10^{-5}$	808.036	852.119
19	0	0	0	5	$3.35 \times 10^{-5}$	694.299	423.708
22	0	0	0	23	$3.35 \times 10^{-5}$	479.359	270.862
23	0	0	0	1	0	0	0
27	0	0	0	27	0	0	0

Fig. 7. IEEE-33 voltage profile for  $s_u = 20$ .Fig. 8. IEEE-33 voltage profile for  $s_u = 50$ .

The analysis so far examines the sole role of each node to the best siting and sizing for the DG units since it is not yet clear whether these prioritized nodes define optimal solutions pairing with each other, or with less frequent nodes. Therefore, an additional data assessment approach has been performed aiming to highlight the co-existence of the most frequent combinations of nodes in all resulted solutions. The existence of all possible combinations, among network's nodes, up to nine different nodes within the 8000 solutions is examined and is described in (18). This number has been selected, based on the performed simulations, since none solution resulted in higher than nine DG units to be installed.

$$\binom{33}{n} \in \left\{ s_i, i = 1 \dots, 2000 \text{ for } s_u = 20 \right\} \text{ with } n = 1, \dots, 9 \quad (18)$$

$$\left\{ s_i, i = 1 \dots, 6000 \text{ for } s_u = 50 \right\}$$

where  $s_i$  is the solution of the snapshot consisting of the optimal DG number with their installation nodes and optimal DG power output. Thus, the simultaneous existence of up to nine nodes to the solutions of all examined snapshots is scanned in order to determine the appearance frequency of all possible combinations. The results from this analysis are summarized in Table 5. In Table 5, A1–A9 refer to combinations from (18) for  $s_u = 20$  and  $n = 1, \dots, 9$ , while B1–B9 refer to combinations from (18) for  $s_u = 50$  and  $n = 1, \dots, 9$ , respectively.

In order to identify the optimal DG number for the best fixed solution the following scenarios are examined:

- Scenario1 (SC#1): Each of the combinations A1–A9 and B1–B9, is considered as a candidate fixed solution. This solution has been applied to all snapshots with different load compositions,

**Table 5**  
Dominant combinations.

	Nodes in combination	Appearance frequency (%)
A1	30	98.46
A2	14, 30	87.71
A3	7, 14, 30	61.29
A4	7, 14, 25, 30	33.38
A5	3, 7, 14, 25, 30	21.54
A6	3, 7, 8, 14, 25, 30	4.73
A7	2, 3, 7, 8, 14, 25, 30	1.19
A8	3, 6, 10, 14, 25, 30, 31, 33	0.80
A9	2, 3, 4, 7, 8, 14, 18, 25, 30	0.05
B1	30	91.53
B2	14, 30	75.41
B3	7, 14, 30	44.16
B4	3, 14, 25, 30	25.53
B5	3, 7, 14, 25, 30	15.33
B6	3, 7, 8, 14, 25, 30	3.55
B7	3, 6, 8, 11, 14, 25, 30	0.55
B8	3, 4, 7, 8, 14, 21, 25, 30	0.28
B9	2, 3, 6, 9, 15, 21, 24, 30, 32	0.02

although it is highly possible to be different from the optimal one, in order to compute the energy loss reduction.

- Scenario2 (SC#2): Alternative to A1–A9 and B1–B9 combinations, the a1–a9 and b1–b9 have been formed based on the prioritization list regarding the node criticality that is presented in Table 4. For example, for A6 combination the fixed solution based on Table 5 is 3, 7, 8, 14, 25, 30 while the respective one, i.e., a6, based on Table 4 is 3, 7, 14, 24, 25, 30 (first six highly prioritized nodes).

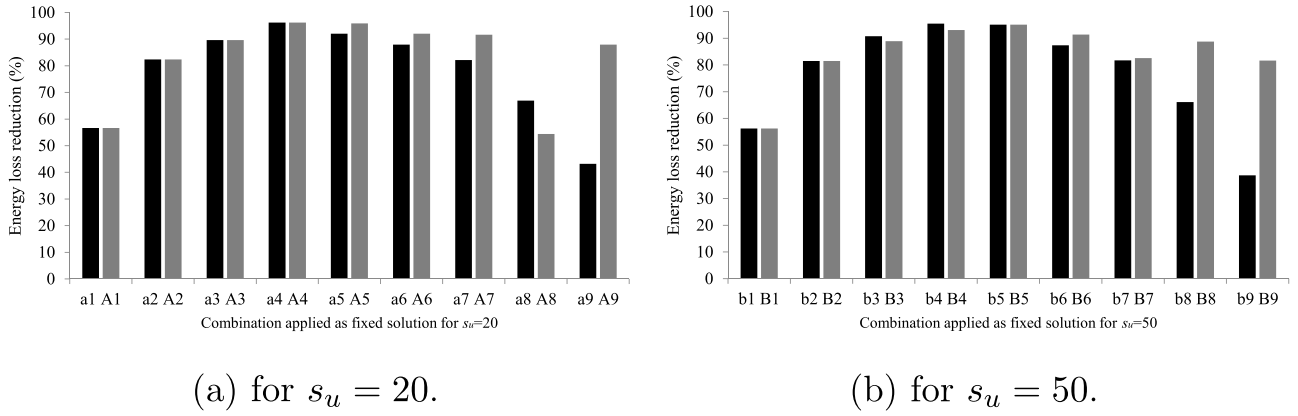


Fig. 9. SC#1 and SC#2.

**Table 6**  
Node criticality for the examined snapshots.

	Best fixed solution	Critical node and the respective P–Q output (kW–kVAr)				Energy loss reduction
$s_u = 20$	A4	7 (755–323)	14 (654–279)	25 (775–341)	30 (863–862)	96.20%
	a4	7 (755–323)	14 (654–279)	25 (775–341)	30 (863–862)	96.20%
$s_u = 50$	B5	3 (92–286)	7 (753–319)	14 (641–274)	25 (815–337)	95.11%
	b4		7 (753–319)	14 (641–274)	25 (815–337)	95.48%

Rationally, the combination with the maximum energy loss reduction is considered as the one to define the optimal number concerning the nodes to host DG units for the final fixed solution. In Fig. 9a and b the efficiency of the fixed solutions by SC#1 and SC#2 regarding the energy loss reduction is presented. For  $s_u = 20$ , the best fixed solution, resulted either by SC#1 or SC#2 approach, i.e. a  $Ai$  or  $ai$  solution, refers to the same four specific installation points (7, 14, 25, 30) with energy loss reduction over than 90%. For  $s_u = 50$ , the best fixed solution for SC#2, i.e. a  $Bi$  or  $bi$  solution, is the same as stated earlier, i.e., nodes 7, 14, 25, 30. If the best fixed solution outcome is based on SC#1, then this solution concerns five installation nodes, i.e. 3, 7, 14, 25, 30. Among these two solutions the best is the one with four installation points as indicated by Fig. 9a and b, i.e. 7, 14, 25, 30. Therefore, the results in Fig. 9a and b suggest that for load variations up to  $\pm 50\%$  from the average load composition of the DN, the best fixed solution regarding the siting and sizing of DG units that could yield the highest energy loss reduction involves the most critical nodes (the first four) that

have resulted by the proposed methodology. Table 6 summarizes the best fixed solutions, among all examined candidate ones, as presented in Fig. 9a and b.

The present analysis proposes an off-line tool for the evaluation of a near optimal solution that could yield satisfying energy loss reduction despite the load variations, and at the same time this solution could exploit the benefits of fixed installation points with fixed power output.

Two basic conclusions are derived from the results shown in Table 6: the capability of finding a fixed solution in terms of installation nodes under different load compositions and the power output of the respective DG units to be installed in the critical nodes. It seems that despite the variations range, the power output for the DG units is approximately the same for three out of the four critical nodes. Only for node 25, a capacity difference of 40 kW of active power exists between intense load variations, i.e.,  $s_u = 50$ , in respect to smoother ones, i.e.,  $s_u = 20$ .

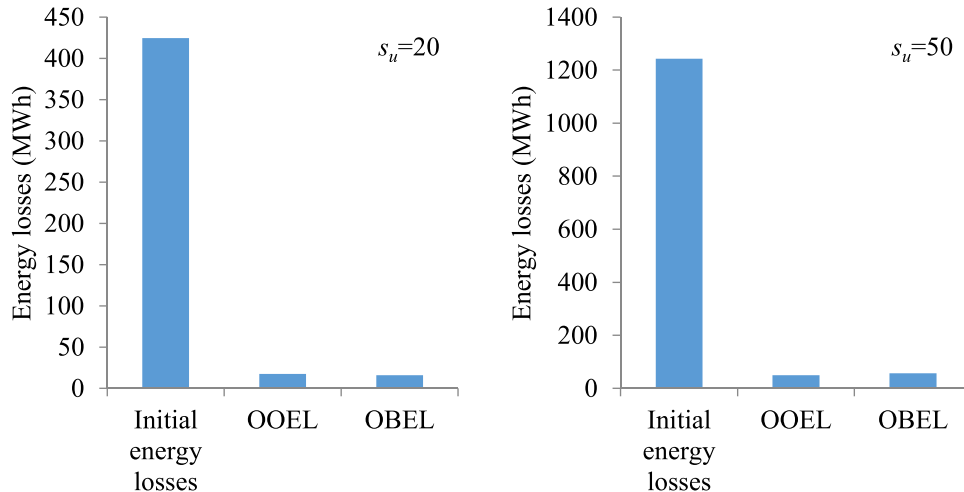


Fig. 10. Performance evaluation of fixed solution in respect to optimal.



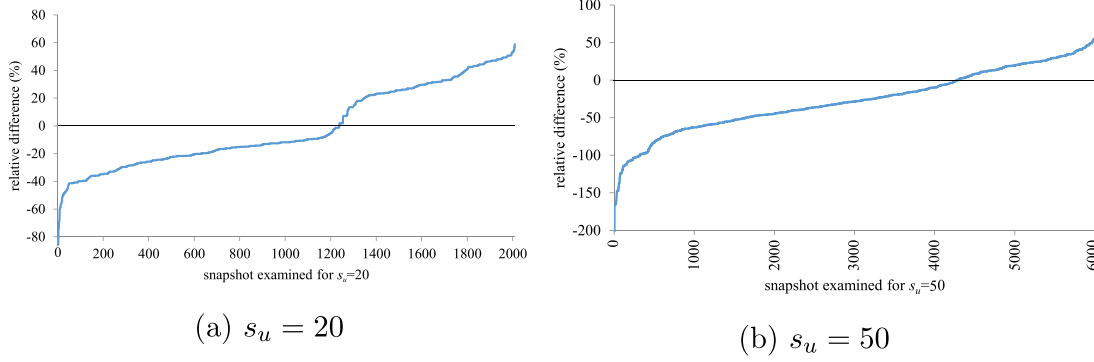


Fig. 11. Performance of fixed solution.

#### 4.2. Proposed solution evaluation

In order to evaluate the efficiency of the fixed solution highlighted in Fig. 9a and b and Table 6, the following assumptions have been made. For every snapshot with time duration  $\Delta t$  in which the load composition is assumed constant, an optimal solution for the ODGP problem provided by the LPSOV algorithm is considered to yield energy losses expressed as:

$$OEL = OPL \cdot \Delta t \quad (19)$$

where  $OEL$  is the Optimal Energy Losses for duration  $\Delta t$ ,  $OPL$  is the Optimal Power Losses for the constant load composition after the DG penetration resulted by the ODGP problem, and  $\Delta t$  is the duration of hours of which the snapshot refers to constant load composition. The Overall Optimal Energy Losses ( $OOEL$ ) for the whole time period is computed by the following expression:

$$OOEL = \sum_{\Delta t=1}^k OEL_k \quad (20)$$

where  $k = 1, \dots, 2000$  for  $s_u = 20$  and  $k = 1, \dots, 6000$  for  $s_u = 50$ .

The best fixed solution resulted by the proposed methodology for every snapshot, as presented in Table 6, i.e. solution A4 or a4 since they are the same in terms of optimal DG siting and sizing for  $s_u = 20$  and b4 for  $s_u = 50$ , is applied as a candidate ODGP best solution. Thus, the Best Energy Losses ( $BEL$ ) caused by the penetration of DG units indicated by this solution is expressed as follows:

$$BEL = BPL \cdot \Delta t \quad (21)$$

where  $BEL$  is the Best Energy Losses for duration  $\Delta t$ ,  $BPL$  is the Best Power Losses for the constant load composition after the DG penetration resulted by the ODGP problem, and  $\Delta t$  is the duration in hours for which the snapshot refers to constant load composition.

The Overall Best Energy Losses ( $OBEL$ ) for the whole time period is computed by the following expression:

$$OBEL = \sum_{\Delta t=1}^k BEL_k \quad (22)$$

Based on the results shown in Fig. 10, the concept of critical nodes for the ODGP problem regarding energy loss reduction is highly supported since it is proved that:

- For  $s_u = 20$ , the proposed fixed solution yields almost the same energy loss reduction to the aggregation of the 2000 optimal solutions. The difference between them is approximately 0.3% in respect to the initial losses without DG penetration.
- For  $s_u = 50$ , the proposed fixed solution is very close to the optimal energy loss reduction by the aggregation of 6000 optimal solutions. In this case, the  $OOEL$  solution is better than the  $OBEL$  one by 0.56% in comparison to the initial losses.

Finally, in order to evaluate the local performance of the best fixed solution for each snapshot, when compared to the local respective optimal solution as provided by the LPSOV application, the relative difference between these two solutions is expressed in (14) and presented in Figs. 7 and 8 for  $s_u = 20$  and  $s_u = 50$  respectively.

$$\text{Relative Difference}_k = \frac{OEL_k - BEL_k}{OEL_k} \quad (23)$$

As easily observed, Fig. 11a and b, intend to show the performance of the proposed fixed solution for every examined snapshot. More specific, the performance of the solution by the LPSO for every snapshot is compared to the fixed one proposed by the authors' methodology. These results present the trade-off regarding the

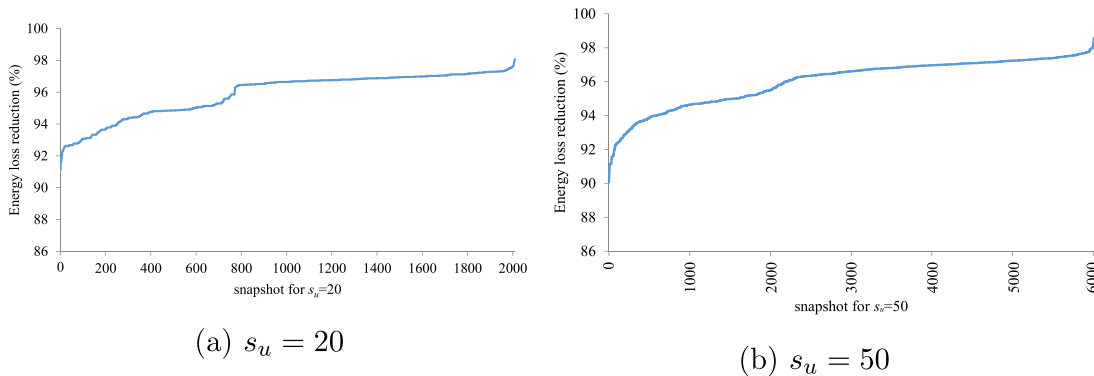


Fig. 12. Efficiency of solutions provided by LPSOV algorithm.

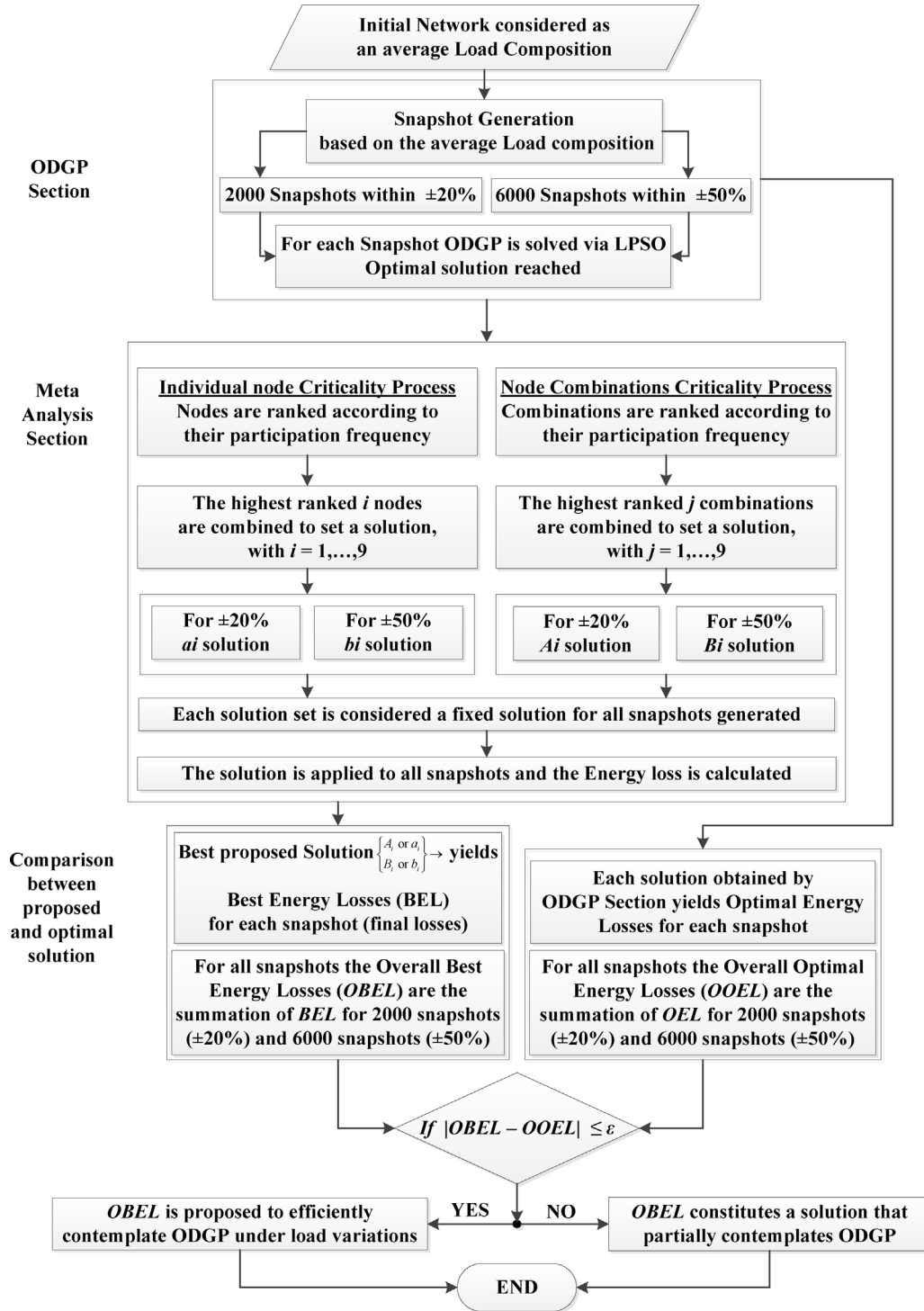


Fig. 13. Flowchart of the proposed methodology.

energy loss reduction when the fixed solution is applied regardless the load composition of the distribution network. In some cases, this fixed solution is better than the considered optimal one by the LPSO, while in others, it is slightly worse. It should be reminded that each optimal local solution has been selected among 50 given solutions after repetitive applications of the algorithm. Since the LPSOV algorithm is a heuristic algorithm, the implementation cannot guarantee by definition the exact optimal solution. Nevertheless, the solution is highly possible to be a near optimal one and this is verified by the results since for all examined snap-

shots, the LPSOV algorithm provides solutions that yield energy loss reduction above 91% for  $s_{ll} = 20$ , and above 90% for  $s_{ll} = 50$  as illustrated in Fig. 12a and b. In these figures the high performance of the LPSOV algorithm is evident since the worst performance refers to the significant high loss reduction of approximately 90%.

A complete flowchart that summarizes the proposed methodology is presented in Fig. 13. The flowchart consists of 3 parts: The upper part concerns the optimal solution of the ODPG problem for the snapshots formed, in order to simulate the DN's load variations. The intermediate part refers to the results assessment in order to

define the node criticality ranking. Finally, the lowest part presents the evaluation of the proposed solutions in terms of overall energy loss reduction, in order to highlight the efficiency of possible fixed solutions that rely on the nodes criticality concept.

## 5. Conclusions

In this paper, the impact of load variations on the ODGP problem for energy loss reduction is examined. Initially, load variations are modeled by considering numerous snapshots of the DN, examined with different load compositions that have resulted stochastically under a uniform distribution approach. The base case scenario considers the average load composition of the DN, while smooth and intense load alterations have properly been considered. The first part of the analysis shows that for every altered load composition, the optimal siting and sizing of the DG units to be installed, as resulted by a LPSOV algorithm, is different. The energy loss minimization in such a case should be subject to various installation points for the DG units and controllable power output.

The second part of the analysis proposes a methodology aiming to identify the most critical nodes of the DN that play a crucial role to the ODGP problem. Each node's criticality is quantified to form a prioritization list for DG installation. This methodology aims at investigating whether a fixed solution consisting of the most critical nodes could be considered as a high efficient solution with near-optimal energy loss reduction. The results indicate that an efficient fixed ODGP solution can be determined for energy loss reduction regardless the load composition of the network. Such a methodology could constitute a useful guideline for the DNO, regarding the prioritization of candidate DG penetration plans. The benefit here relies on achieving high energy loss reduction under a fixed solution concerning the installation nodes and the power output of the DG units.

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