Γενικά Μαθηματικά
Σειρές (χωρίς λόγια)

Δρ Γεώργιος Μενεξές
Επίκουρος Καθηγητής

Αριστοτέλειο Πανεπιστήμιο Θεσσαλονίκης, Σχολή Γεωπονίας, Δασολογίας και Φυσικού Περιβάλλοντος, Τμήμα Γεωπονίας
Εργαστήριο Γεωργίας

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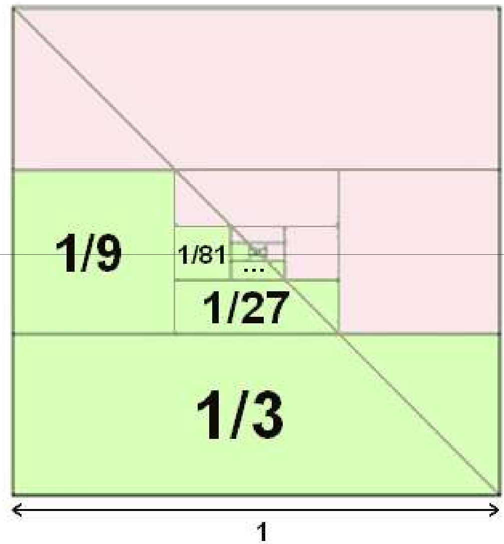
@ArtOfMathematics

Without words

$$\sum_{k=1}^{+\infty} \frac{1}{3^k} =$$



$$= \frac{1}{2}$$

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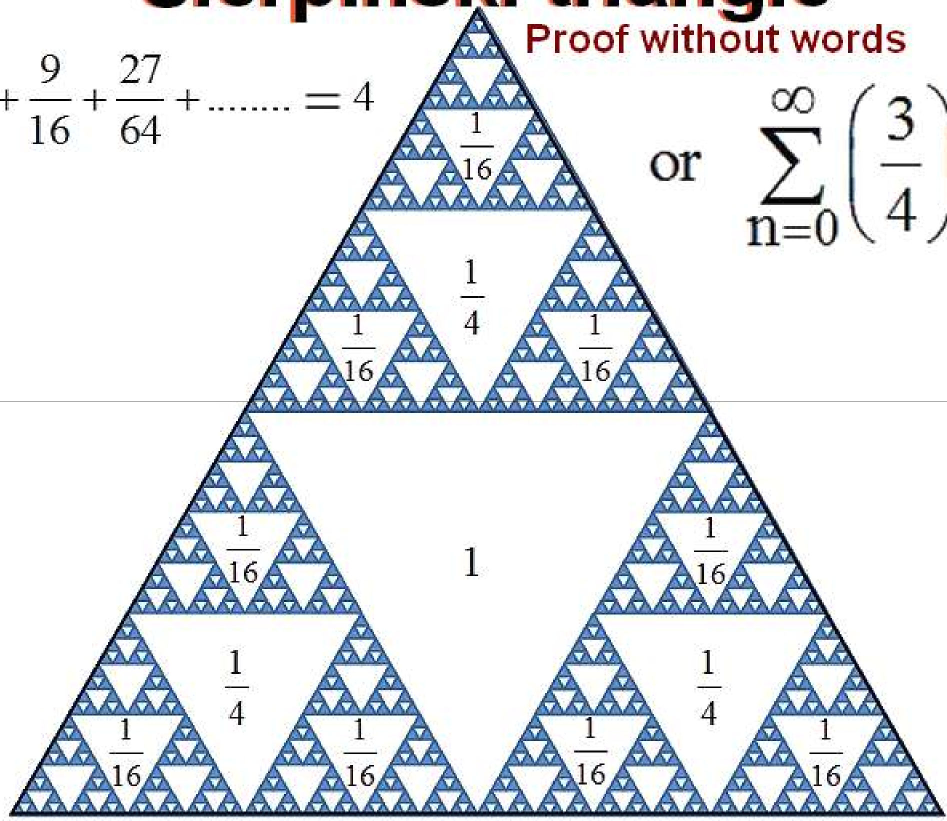
Sierpinski triangle

Proof without words

$$1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+........=4$$

or

$$\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{n}=4$$



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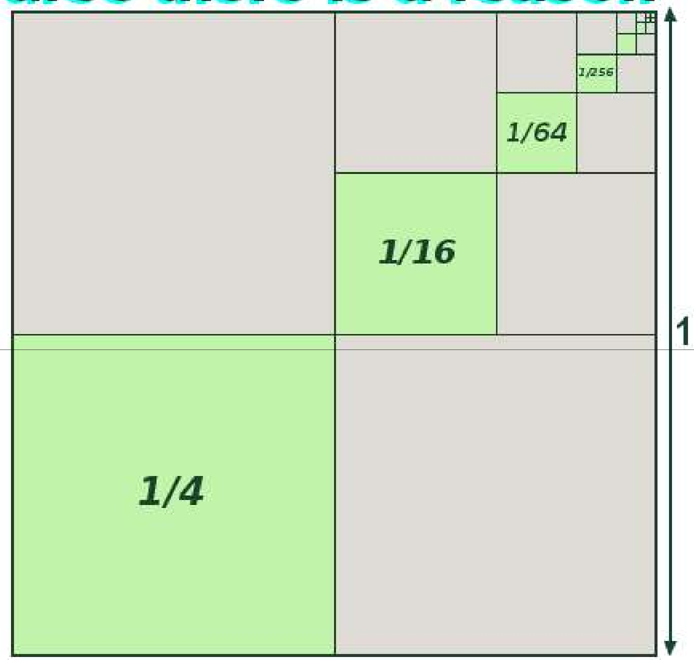
We called it a geometric sequence, of course there is a reason

$$\sum_{k=1}^{+\infty} \frac{1}{4^k} = \frac{1}{3}$$



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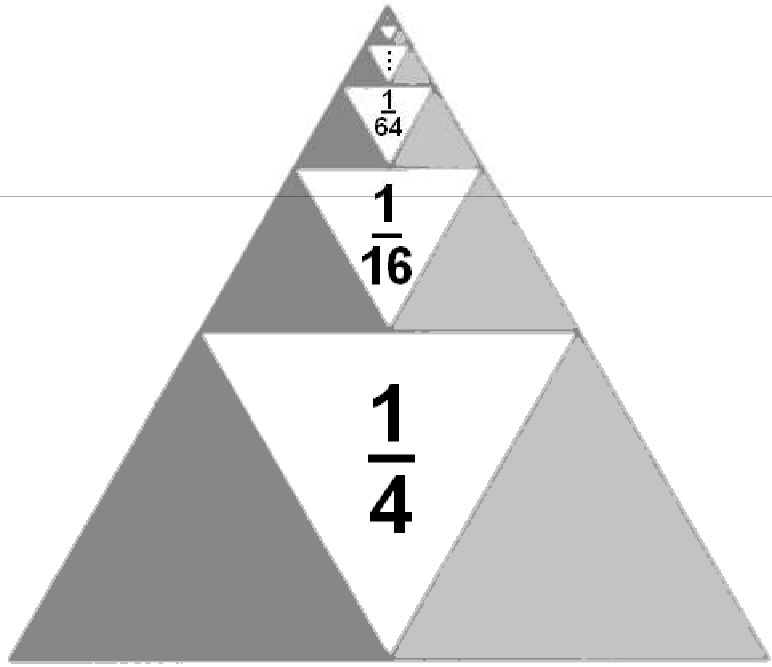
Proof without words

$$\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots = \frac{1}{3}$$



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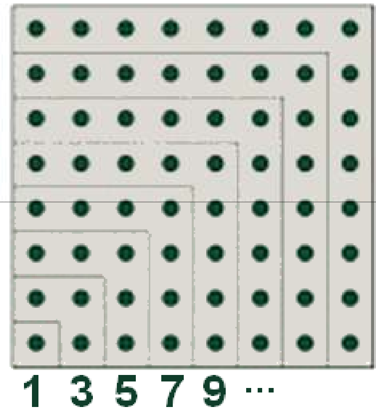
Without words

$$\sum_{k=1}^{n}(2k - 1) = \text{[diagram]} = n^2$$



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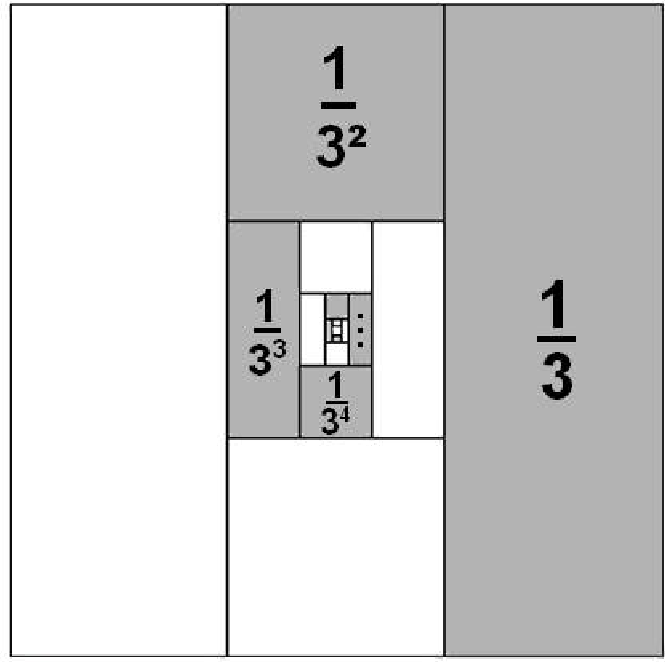
Without Words



$$\frac{1}{2} = \sum_{i=1}^{\infty} \frac{1}{3^i} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \dots$$

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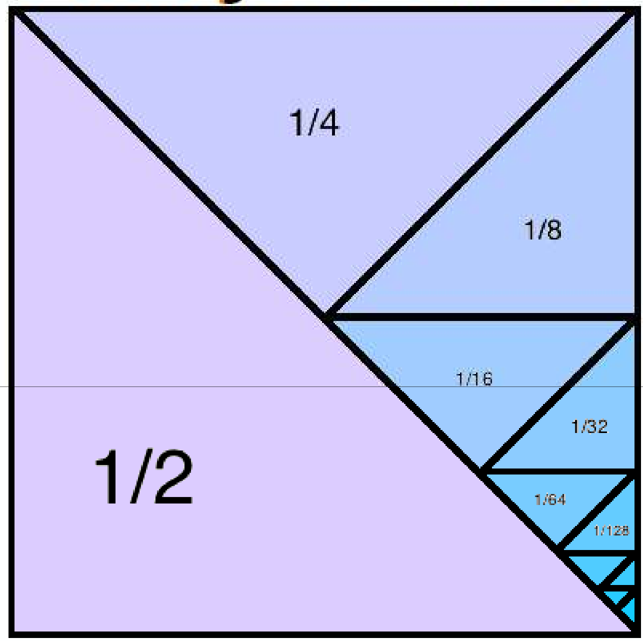
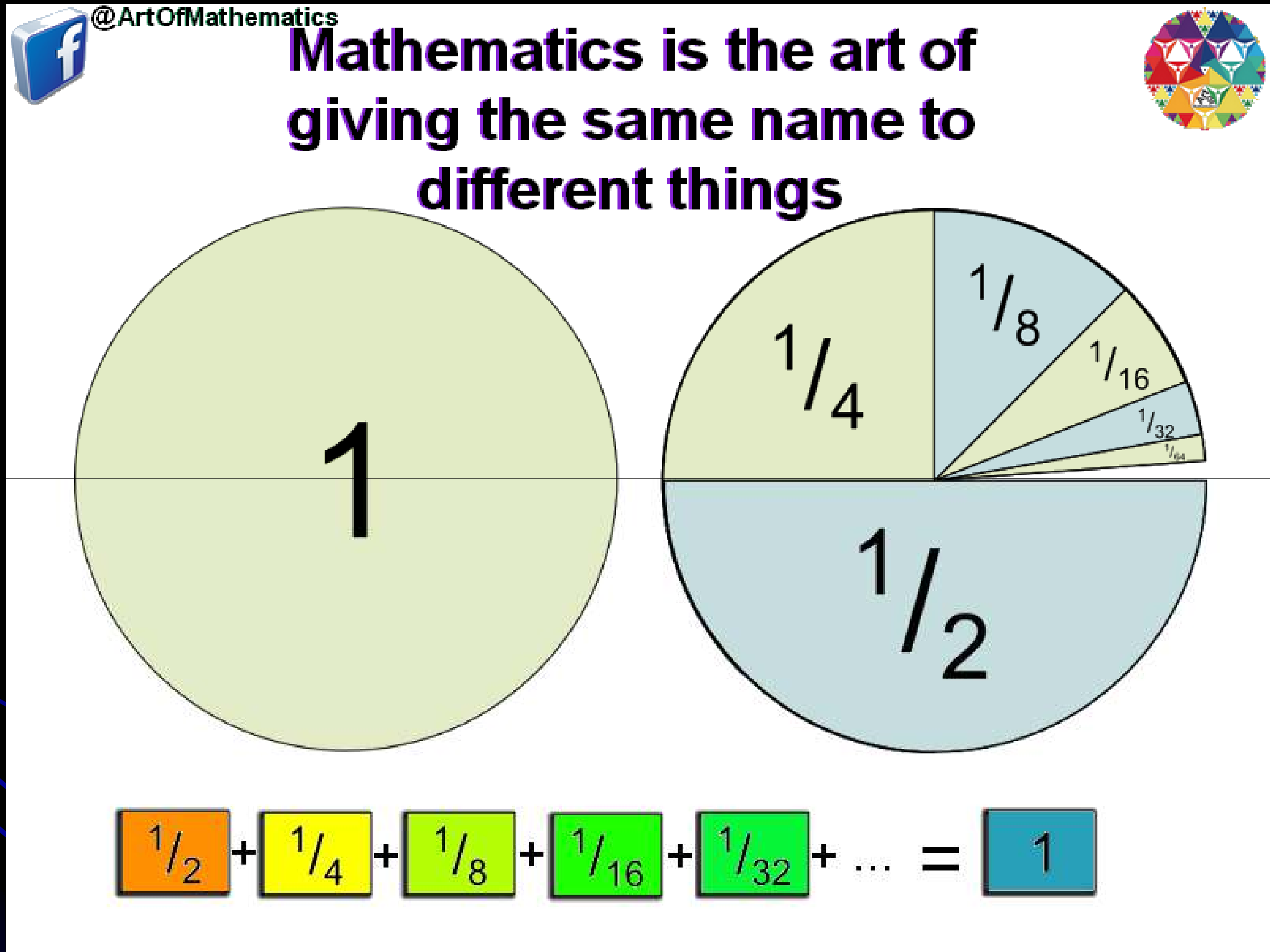
Link your ideas



$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 1$$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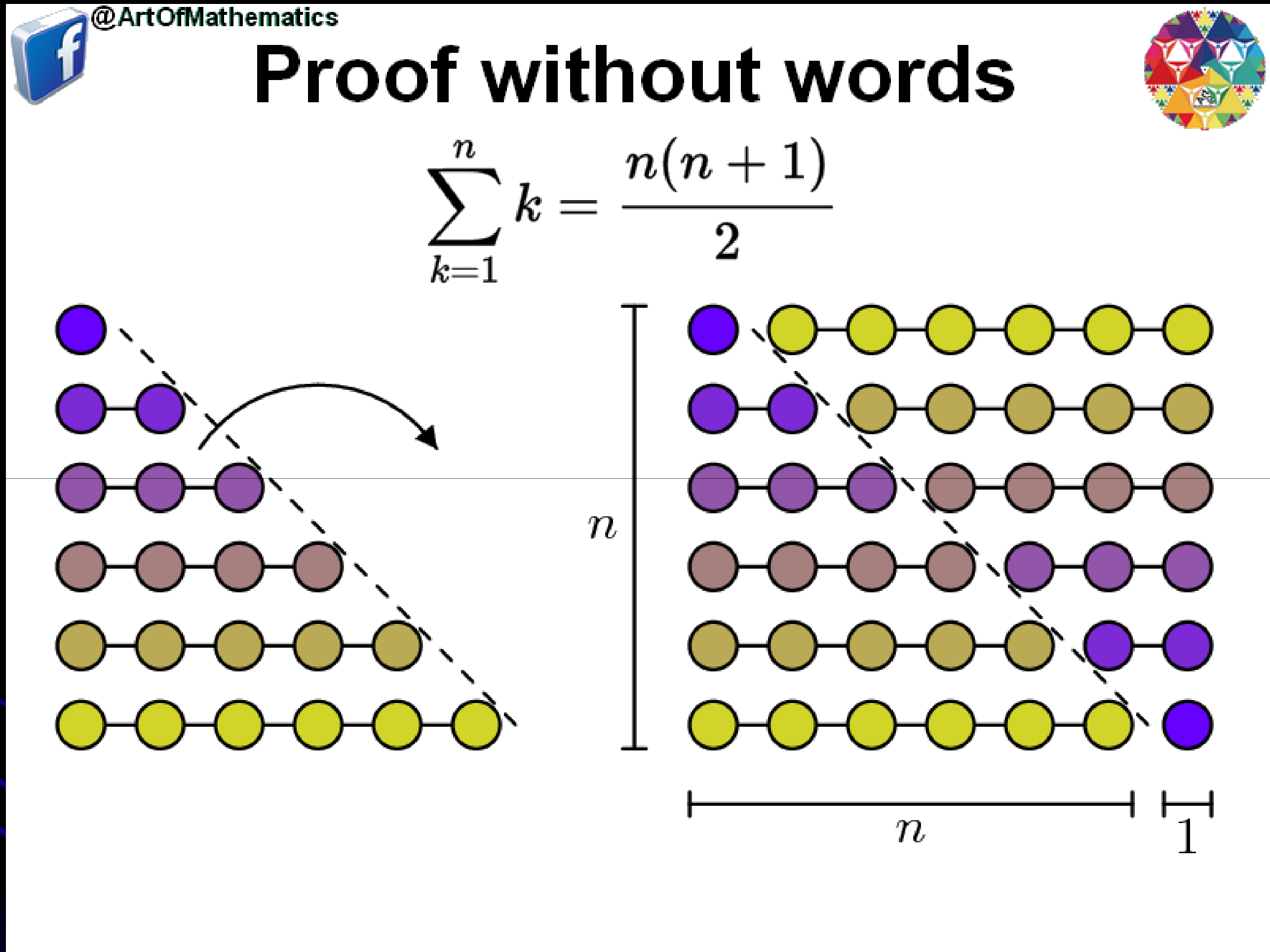
Mathematics is the art of giving the same name to different things
1
1/4
1/8
1/16
1/32
1/2
1/2 + 1/4 + 1/8 + 1/16 + 1/32 + … = 1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Proof without words
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
n
n
1

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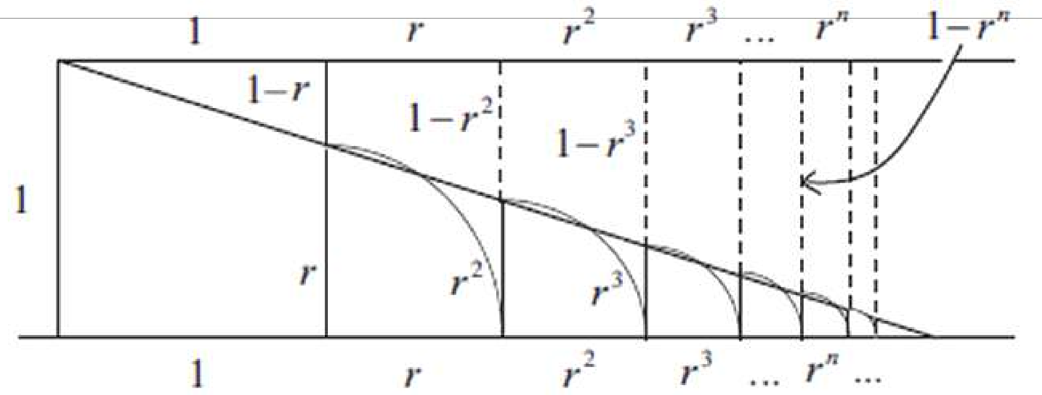
Proof without words

Partial sum and sum of a geometric series

Let r be a positive real number such that $0 < r < 1$, then:

$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}; \qquad \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$



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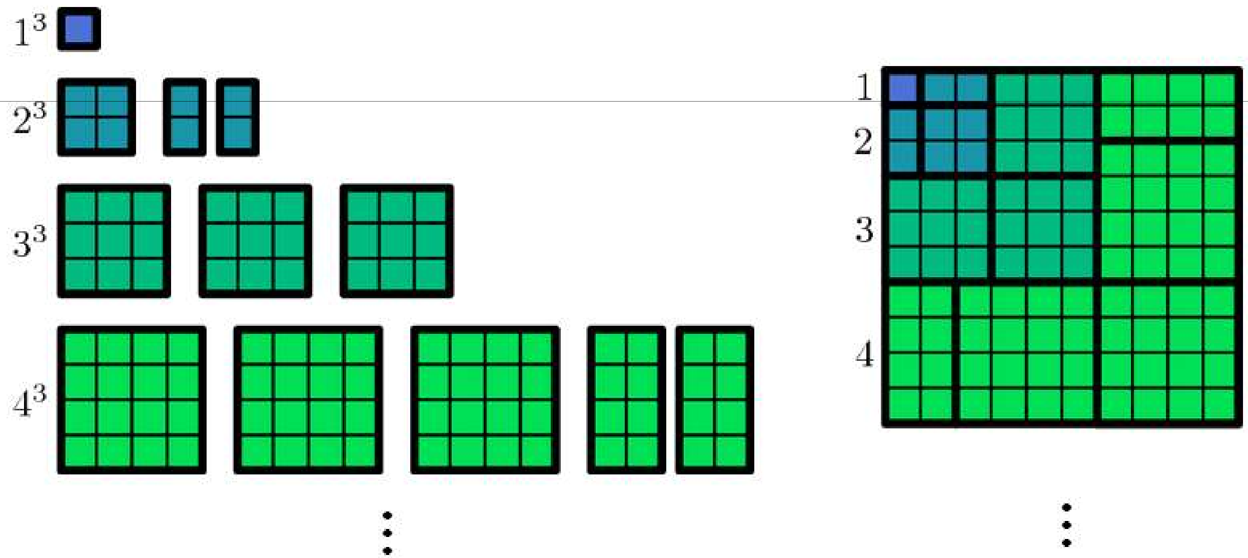
Proof without words

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$



Art Arithmetic Thinking Of Mathematics

$$S_n = 1 + 2 + 3 + \dots + n-2 + n-1 + n$$

Or

$$S_n = n + n-1 + n-2 + \dots + 3 + 2 + 1$$

$$S_n + S_n = n+1 + n-1+2 + \dots + n+1$$

$$2S_n = \underbrace{n+1 + n+1 + n+1 + \dots + n+1}_{n \text{ terms}}$$

$$2S_n = n(n+1)$$

$$S_n = \frac{n(n+1)}{2}$$

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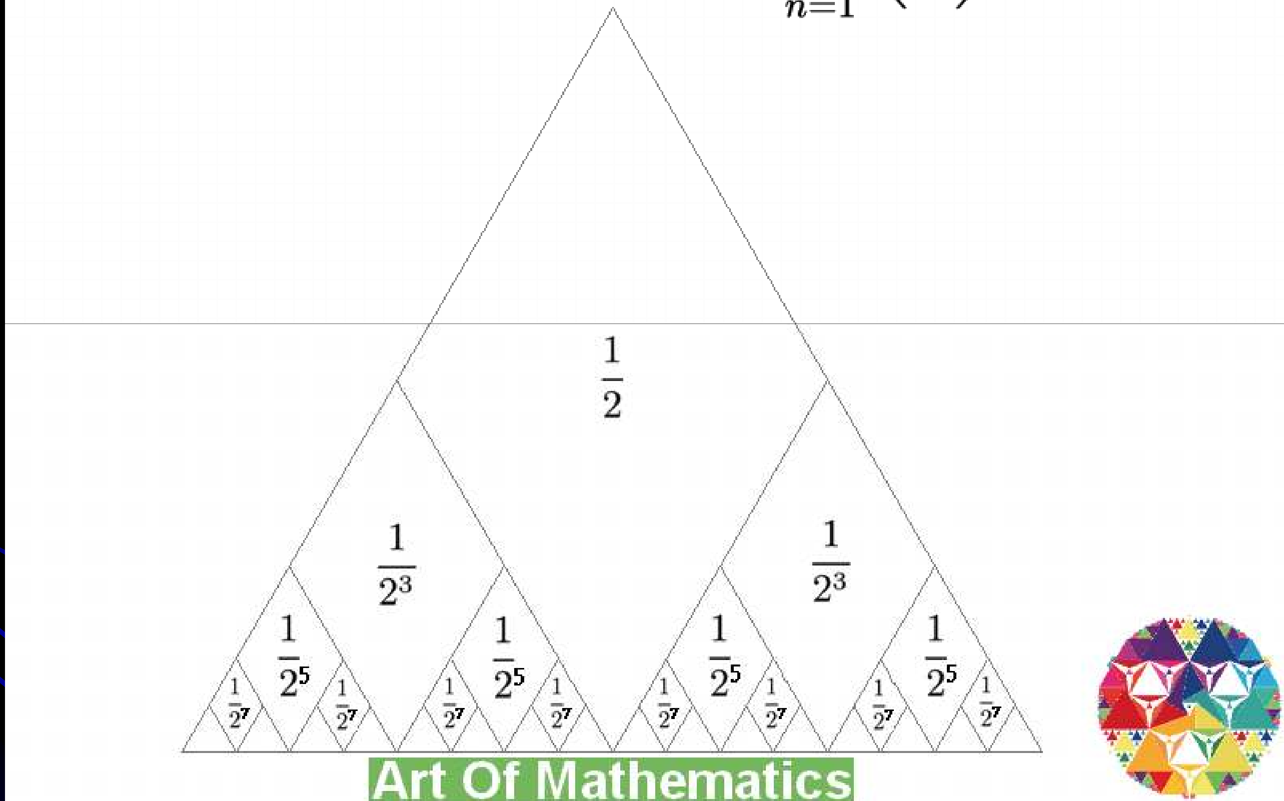
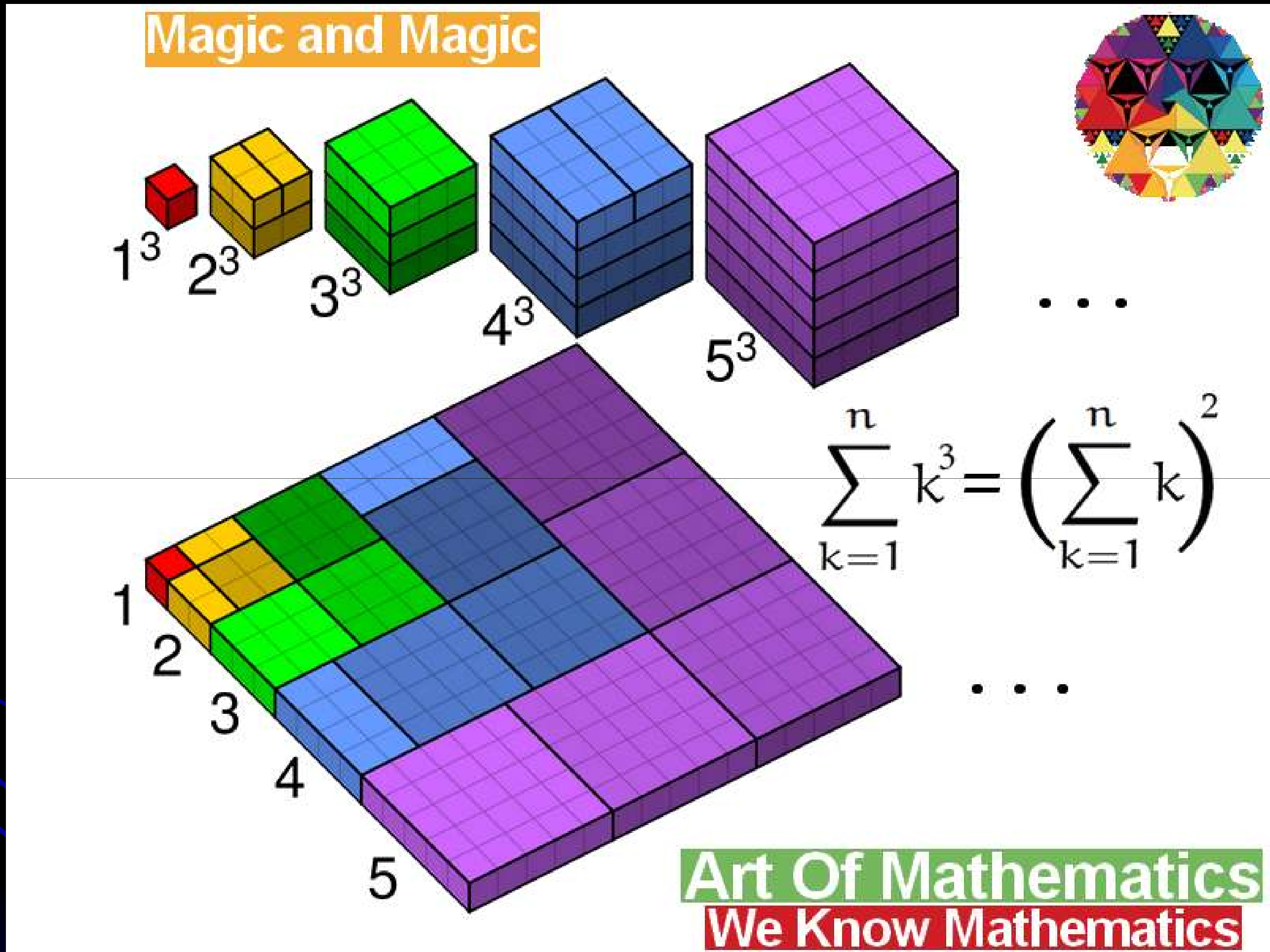
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n}=1$$




Magic and Magic
1³ 2³ 3³ 4³ 5³ . . .
1 2 3 4 5 . . .
∑_{k=1}^{n} k³ = (∑_{k=1}^{n} k)²
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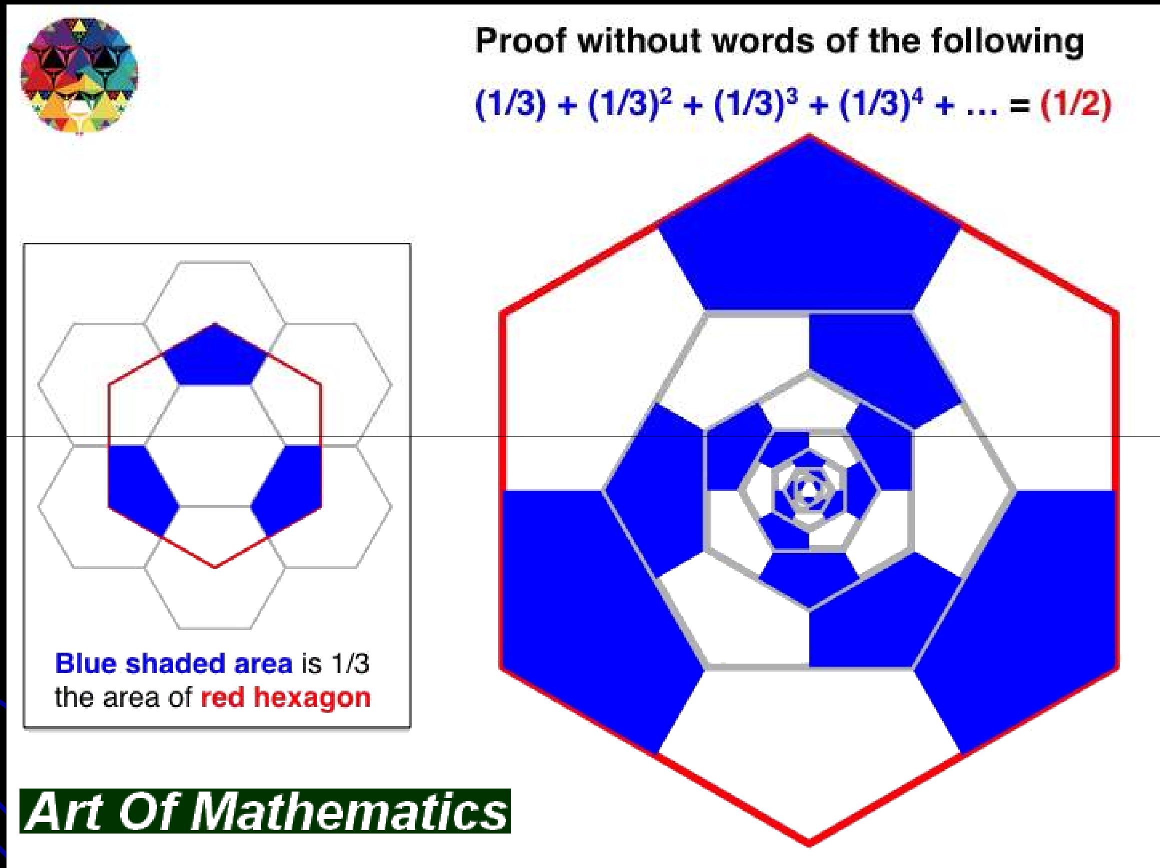
We Know Mathematics


Proof without words of the following
(1/3) + (1/3)^2 + (1/3)^3 + (1/3)^4 + … = (1/2)
Blue shaded area is 1/3 the area of red hexagon
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Οι εικόνες προέρχονται από τη σελίδα στο

Art of Mathematics