



# Split-Split Plot Arrangement

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## Remarks (I)

The split-split plot arrangement is especially suited for three or more factor experiments where different levels of precision are required for the factors evaluated.

This arrangement is characterized by:

1. Three plot sizes corresponding to the three factors; namely, the largest plot for the main factor, the intermediate size plot for the subplot factor, and the smallest plot for the sub-subplot factor.
2. There are three levels of precision with the main plot factor receiving the lowest precision, and the sub-subplot factor receiving the highest precision.

## Remarks (II)

Split-split plot arrangement randomized as an RCBD. Three levels of the whole plot factor, A, two levels of the subplot factor, B, and three levels of the sub-subplot factor, C. Diagram shows the first replicate.

$a_0$ Whole plot	$a_1 b_0$ subplots	$a_2 b_1 c_0$	$a_2 b_1 c_2$	$a_2 b_1 c_1$
	$a_1 b_1$ subplots	$a_2 b_0 c_1$	$a_2 b_0 c_0$	$a_2 b_0 c_2$

## Randomization

The randomization procedure for the split-split plot arrangement consists of three parts:

1. Randomly assign whole plot treatments to whole plots based on the experimental design used.
2. Randomly assign subplot treatments to the subplots.
3. Randomly assign sub-subplot treatments to the sub-subplots.

The experimental design used to randomize the whole plots will not affect randomization of the sub and sub-subplots.

## ANOVA Table

The example to be given will be for an RCBD with factor A as the whole plot factor, factor B as the subplot factor, and factor C as the sup-subplot factor. Factors A, B, and C will be considered random effects.

Source of variation	df	Expected mean square
Replicate	r-1	$\sigma^2 + c\sigma^2_\theta + bcc\sigma^2_\gamma + abc\sigma^2_R$
A	a-1	$\sigma^2 + c\sigma^2_\theta + bcc\sigma^2_\gamma + r\sigma^2_{ABC} + rbo\sigma^2_{AC} + rc\sigma^2_{AB} + rbco\sigma^2_A$
Error (a) = RepxA	(r-1)(a-1)	$\sigma^2 + c\sigma^2_\theta + bcc\sigma^2_\gamma$
B	b-1	$\sigma^2 + c\sigma^2_\theta + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + raco\sigma^2_B$
AxB	(a-1)(b-1)	$\sigma^2 + c\sigma^2_\theta + r\sigma^2_{ABC} + rc\sigma^2_{AB}$
Error (b) = RepxB(A)	a(r-1)(b-1)	$\sigma^2 + c\sigma^2_\theta$
C	c-1	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rbo\sigma^2_{AC} + rab\sigma^2_C$
AxC	(a-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC} + rbo\sigma^2_{AC}$
BxC	(b-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$
AxBxC	(a-1)(b-1)(c-1)	$\sigma^2 + r\sigma^2_{ABC}$
Error (c) = RepxC(AxB)	ab(r-1)(c-1)	$\sigma^2$
Total	rabc-1	

## Example

ANOVA of a Split-split Plot Arrangement - Table 1. Data for split-split plot example

Treatments			Replicates				Treatment
A <sub>i</sub>	B <sub>k</sub>	C <sub>l</sub>	1	2	3	4	totals
0	0	0	25.7	25.4	23.8	22.0	96.9
0	0	1	31.8	29.5	28.7	26.4	116.4
0	0	2	34.6	37.2	29.1	23.7	124.6
Subplot tot. Y <sub>000</sub>			92.1	92.1	81.6	72.1	337.9= Y <sub>00</sub>
0	1	0	27.7	30.3	30.2	33.2	121.4
0	1	1	38.0	40.6	34.6	31.0	144.2
0	1	2	42.1	43.6	44.6	42.7	173.0
Subplot tot. Y <sub>001</sub>			107.8	114.5	109.4	106.9	438.6= Y <sub>01</sub>
Whole plot tot. Y <sub>00..</sub>			199.9	206.6	191.0	179.0	776.5= Y <sub>0..</sub>
1	0	0	28.9	24.7	27.8	23.4	104.8
1	0	1	37.5	31.5	31.0	27.8	127.8
1	0	2	38.4	32.5	31.2	29.8	131.9
Subplot tot. Y <sub>100</sub>			104.8	88.7	90.0	81.0	364.5= Y <sub>10..</sub>
1	1	0	38.0	31.0	29.5	30.7	129.2
1	1	1	36.9	31.9	31.5	35.9	136.2
1	1	2	44.2	41.6	38.9	37.6	162.3
Subplot tot. Y <sub>111</sub>			119.1	104.5	99.9	104.2	427.7= Y <sub>11..</sub>

## Example (cont.)

Whole plot tot. $Y_{i..}$	223.9	193.2	189.9	185.2	792.2= $Y_{1..}$
2 0 0	23.4	24.2	21.2	20.9	89.7
2 0 1	25.3	27.7	23.7	24.3	101.0
2 0 2	29.8	29.9	24.3	23.8	107.8
Subplot tot. $Y_{i20}$	78.5	81.8	69.2	69.0	298.5= $Y_{20}$
2 1 0	20.8	23.0	25.2	23.1	92.1
2 1 1	29.0	32.0	26.5	31.2	118.7
2 1 2	36.6	37.8	34.8	40.2	149.4
Subplot tot. $Y_{i21}$	86.4	92.8	86.5	94.5	360.2= $Y_{21}$
Whole plot tot. $Y_{i2..}$	164.9	174.6	155.7	163.5	658.7= $Y_{2..}$
Rep total $Y_{i..}$	588.7	574.4	536.6	527.7	2227.4= $Y_{...}$

## Preliminary Preparation

Table 2. Totals for two-way interactions.

A X B ( $Y_{ijk}$ )		A X C ( $Y_{ijl}$ )			B X C ( $Y_{klj}$ )			
b <sub>0</sub>	b <sub>1</sub>	c <sub>0</sub>	c <sub>1</sub>	c <sub>2</sub>	b <sub>0</sub>	b <sub>1</sub>		
a <sub>0</sub>	337.9	438.6	218.3	260.6	297.6	c <sub>0</sub>	291.4	342.7
a <sub>1</sub>	364.5	427.7	234.0	264.0	294.2	c <sub>1</sub>	345.2	399.1
a <sub>2</sub>	298.5	360.2	181.8	219.7	257.2	c <sub>2</sub>	364.3	484.7

Table 3. Totals for main effects.

A ( $Y_{i..}$ )			B ( $Y_{.kj}$ )		C ( $Y_{.lj}$ )		
a <sub>0</sub>	a <sub>1</sub>	a <sub>2</sub>	b <sub>0</sub>	b <sub>1</sub>	c <sub>0</sub>	c <sub>1</sub>	c <sub>2</sub>
776.5	792.2	658.7	1000.9	1226.5	634.1	744.3	849.0

# Calculations (1)

Step 1. Calculate correction factor

$$CF = \frac{Y_{...}^2}{rabc}$$

$$= \frac{(2227.4)^2}{(4 \times 3 \times 2 \times 3)}$$

$$= 68,907.094$$

Step 2. Calculate total sum of squares

$$\text{Total SS} = \sum Y_{ijkl}^2 - CF$$

$$\text{Total SS} = (25.7^2 + 25.4^2 + 23.8^2 + \dots + 40.2^2) - CF = 2840.606$$

# Calculations (2)

Step 3. Calculate replicate sum of squares

$$\text{Rep SS} = \frac{\sum Y_{i...}^2}{abc} - CF$$

$$= \frac{(588.7^2 + 574.4^2 + 536.6^2 + 527.7^2)}{3 \times 2 \times 3} - CF$$

$$= 143.456$$

Step 4. Calculate A sum of squares.

$$\text{A SS} = \frac{\sum Y_{j...}^2}{rbc} - CF$$

=

$$= \frac{(776.5^2 + 792.2^2 + 658.7^2)}{4 \times 2 \times 3} - CF$$

$$= 443.689$$

## Calculations (3)

Step 5. Calculate Whole plot sum of squares.

$$\begin{aligned}\text{Whole Plot SS} &= \frac{\sum Y_{ij}^2}{bc} - CF \\ &= \frac{(199.9^2 + 206.6^2 + 191.0^2 + \dots + 163.5^2)}{bc} - CF \\ &= 698.903\end{aligned}$$

Step 6. Calculate Error(a) sum of squares.

$$\begin{aligned}\text{Error (a) SS} &= \text{Whole plot SS} - A \text{ SS} - \text{Rep SS} \\ &= 698.903 - 443.689 - 143.456 = 111.749\end{aligned}$$

## Calculations (4)

Step 7. Calculate B sum of squares.

$$\begin{aligned}B \text{ SS} &= \frac{\sum Y_{..k}^2}{rac} - CF \\ &= \frac{(1000.9^2 + 1226.5^2)}{4 \times 3 \times 3} - CF \\ &= 706.880\end{aligned}$$

Step 8. Calculate A X B sum of squares.

$$\begin{aligned}AxB \text{ SS} &= \frac{\sum Y_{jk}^2}{rc} - CF - A \text{ SS} - B \text{ SS} \\ &= \frac{(337.9^2 + 364.5^2 + 298.5^2 + \dots + 438.6^2)}{4 \times 3} - CF - A \text{ SS} - B \text{ SS} \\ &= 40.687\end{aligned}$$

## Calculations (5)

Step 9. Calculate subplot sum of squares.

$$\begin{aligned}\text{Subplot SS} &= \frac{\sum Y_{ijk}^2}{c} - CF \\ &= (92.1^2 + 92.1^2 + 81.6^2 + \dots + 94.5^2) / 3 - CF \\ &= 1524.813\end{aligned}$$

Step 10. Calculate Error(b) sum of squares.

$$\begin{aligned}\text{Error(b) SS} &= \text{Subplot SS} - A \times B \text{ SS} - B \text{ SS} - \text{Error(a) SS} - A \text{ SS} - \text{Rep SS} \\ &= 1524.813 - 40.687 - 706.88 - 111.749 - 443.689 - 143.465 = 78.343\end{aligned}$$

## Calculations (6)

Step 11. Calculate C sum of squares.

$$\begin{aligned}C \text{ SS} &= \frac{\sum Y_{.l}^2}{rab} - CF \\ &= \frac{(634.1^2 + 744.3^2 + 849.0^2)}{4 \times 3 \times 2} - CF \\ &= 962.335\end{aligned}$$

Step 12. Calculate A X C sum of squares.

$$\begin{aligned}A \times C \text{ SS} &= \frac{\sum Y_{.l}^2}{rb} - CF - A \text{ SS} - C \text{ SS} \\ &= \frac{(218.3^2 + 234.0^2 + 181.3^2 + \dots + 257.2^2)}{4 \times 2} - CF - A \text{ SS} - C \text{ SS} \\ &= 13.1097\end{aligned}$$

## Calculations (7)

Step 13. Calculate B X C sum of squares.

$$\begin{aligned} \text{BxCSS} &= \frac{\sum Y_{\cdot k l}^2}{ra} - CF \cdot BSS - CSS \\ &= \frac{(291.4^2 + 345.2^2 + 364.3^2 + \dots + 484.7^2)}{4 \times 3} - CF \cdot BSS - CSS \\ &= 127.831 \end{aligned}$$

## Calculations (8)

Step 14. Calculate AxBxC sum of squares.

$$\begin{aligned} \text{AxBxCSS} &= \frac{\sum Y_{jkl}^2}{r} - CF \cdot ASS - BSS - CSS - AxBSS - AxCSS - BxCSS - \\ &= \frac{(96.9^2 + 116.4^2 + 124.6^2 + \dots + 149.4^2)}{4} - CF \cdot ASS - BSS - CSS - AxBSS - AxCSS - BxCSS \\ &= 44.019 \end{aligned}$$

Step 15. Calculate Error(c) sum of squares.

$$\begin{aligned} \text{Error(c) SS} &= \text{Total SS} - \text{AxBxC SS} - \text{BxC SS} - \text{AxC SS} - \text{C SS} - \text{Error(b) SS} - \text{AxB SS} - \text{B SS} - \\ &\quad \text{Error(a) SS} - \text{A SS} - \text{Rep SS} \\ &= 168.498 \end{aligned}$$

## Calculations (9)

Step 16 ANOVA

SOV	df	SS	MS	F (A,B, C fixed)
Replicate	3	143.45	47.819	
A	2	443.689	221.844	A MS/Error(a) MS = 11.91**
Error(a)	6	111.749	18.626	
B	1	706.88	706.88	B MS/Error(b) MS = 81.21**
A X B	2	40.688	20.344	AxB MS/Error(b) MS = 2.34
Error(b)	9	78.343	8.705	
C	2	962.335	481.168	C MS/Error(c) MS = 102.80**
A X C	4	13.110	3.277	AxC MS/Error(c) MS = 0.70
B X C	2	127.831	63.915	BxC MS/Error(c) MS = 13.66**
A X B X C	4	44.019	11.005	AxBxC MS/Error(c) MS = 2.35
Error(c)	36	168.498	4.681	
Total	71	2840.606		

## Calculations (10)

### LSD's for Split-split Plot Arrangement

- To compare two whole plot means averaged over all sub and sub-sub-plot treatments (e.g.  $a_0$  vs.  $a_1$ ).

$$LSD = t_{\frac{\alpha}{2}, \text{Err}(a)\text{df}} \sqrt{\frac{2\text{Error}(a)\text{MS}}{rbc}}$$

$$= 2.447 \sqrt{\frac{2(18.626)}{4 \times 2 \times 3}}$$

$$= 3.05$$

## Calculations (11)

2. To compare two subplot means averaged over all whole and sub-sub plot treatments (e.g. b<sub>0</sub> vs. b<sub>1</sub>).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(b)\text{df}} \sqrt{\frac{2\text{Error}(b)\text{MS}}{\text{rac}}}$$
$$= 2.262 \sqrt{\frac{2(8.705)}{4 \times 3 \times 3}}$$
$$= 1.57$$

## Calculations (12)

3. To compare two sub-subplot means averaged over all whole and subplot treatments (e.g. c<sub>0</sub> vs c<sub>1</sub>).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(c)\text{df}} \sqrt{\frac{2\text{Error}(c)\text{MS}}{\text{rab}}}$$
$$= 2.030 \sqrt{\frac{2(4.681)}{4 \times 3 \times 3}} 2$$
$$= 1.27$$

## Calculations (13)

4. To compare two subplot means (averaged over all sub-subplot treatments) at the same levels of the whole plot (e.g. a<sub>0</sub>b<sub>0</sub> vs. a<sub>0</sub>b<sub>1</sub>).

$$LSD = t_{\frac{a}{2}, Err(b) df} \sqrt{\frac{2Error(b)MS}{rc}}$$

$$= 2.262 \sqrt{\frac{2(8.705)}{4 \times 3}}$$

$$= 2.72$$

## Calculations (14)

5. To compare two whole plot means (averaged over all sub-subplot treatments) at the same or different levels of the subplot (e.g. a<sub>0</sub>b<sub>0</sub> vs a<sub>1</sub>b<sub>0</sub> or a<sub>0</sub>b<sub>0</sub> vs a<sub>2</sub>b<sub>1</sub>).

$$LSD = t_{AB} \sqrt{\frac{2[(b-1)Error(b)MS + Error(a)MS]}{rbc}}$$

and

$$t_{AB} = \frac{(b-1)Error(b)MS \left( t_{\frac{a}{2}, Err(b) df} \right) + Error(a)MS \left( t_{\frac{a}{2}, Err(a) df} \right)}{(b-1)Error(b) MS + Error(a) MS}$$

∴

$$t_{AB} = \frac{(2-1)(8.705)(2.262) + 18.626(2.447)}{(2-1)8.705 + 18.626}$$

$$= 2.388$$

and

$$LSD = 2.388 \sqrt{\frac{2[(2-1)8.705 + 18.626]}{4 \times 2 \times 3}}$$

## Calculations (15)

6. To compare two sub-subplot means (averaged over all subplot treatments) at the same levels of the whole plot (e.g.  $a_0c_0$  vs.  $a_0c_1$ ).

$$LSD = t_{\frac{a}{2}, Err(c)df} \sqrt{\frac{2Error(c)MS}{rb}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4x2}}$$

$$= 2.20$$

## Calculations (16)

7. To compare two whole plot means (averaged over all subplot treatments) at the same or different levels of the sub-subplot (e.g.  $a_0c_0$  vs.  $a_1c_0$  or  $a_0c_0$  vs.  $a_2c_1$ ).

$$LSD = t_{AC} \sqrt{\frac{2[(c-1)Error(c)MS + Error(a)MS]}{rbc}}$$

and

$$t_{AC} = \frac{(c-1)Error(c)MS \left( t_{\frac{a}{2}, Err(c)df} \right) + Error(a)MS \left( t_{\frac{a}{2}, Err(a)df} \right)}{(c-1)Error(c)MS + Error(a)MS}$$

∴

$$t_{AC} = \frac{(3-1)(4.681)(2.030) + 18.626(2.447)}{(3-1)4.681 + 18.626}$$

$$= 2.307$$

and

$$LSD = 2.307 \sqrt{\frac{2[(3-1)4.681 + 18.626]}{4x2x3}}$$

$$= 3.52$$

## Calculations (17)

8. To compare two sub-subplot means (averaged over all whole plot treatments) at the same levels of the subplot (e.g.  $b_0c_0$  vs.  $b_0c_1$ ).

$$LSD = t_{\frac{a}{2} Err(c) df} \sqrt{\frac{2 Error(c) MS}{ra}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4 \times 3}}$$

$$= 1.79$$



## Calculations (18)

9. To compare two subplot means (averaged over all whole plot treatments) at the same or different levels of the sub-subplot (e.g.  $b_0c_0$  vs.  $b_1c_0$  or  $b_0c_0$  vs.  $b_2c_1$ ).

$$LSD = t_{BC} \sqrt{\frac{2[(c-1)Error(c)MS + Error(b)MS]}{rac}}$$

and

$$t_{BC} = \frac{(c-1)Error(c)MS \left( t_{\frac{a}{2} Err(c) df} \right) + Error(b)MS \left( t_{\frac{a}{2} Err(b) df} \right)}{(c-1)Error(c)MS + Error(b)MS}$$

∴

$$t_{BC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142 \sqrt{\frac{2[(3-1)4.681 + 8.705]}{4 \times 3 \times 3}}$$

$$= 2.15$$

## Calculations (19)

10. To compare two sub-subplot means at the same combination of whole plot and subplot treatments (e.g.  $a_0b_0c_0$  vs.  $a_0b_0c_2$ ).

$$LSD = t_{\frac{\alpha}{2}, Err(c)df} \sqrt{\frac{2Error(c)MS}{r}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4}}$$

$$= 3.11$$



## Calculations (20)

11. To compare two subplot means at the same level of whole plot and sub-subplot (e.g.  $a_0b_0c_0$  vs.  $a_0b_1c_0$ ).

$$LSD = t_{ABC} \sqrt{\frac{2[(c-1)Error(c)MS + Error(b)MS]}{rc}}$$

and

$$t_{ABC} = \frac{(c-1)Error(c)MS \left( t_{\frac{\alpha}{2}, Err(c)df} \right) + Error(b)MS \left( t_{\frac{\alpha}{2}, Err(b)df} \right)}{(c-1)Error(c)MS + Error(b)MS}$$

∴

$$t_{ABC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142 \sqrt{\frac{2[(3-1)4.681 + 8.705]}{4 \times 3}}$$

$$= 3.72$$

## Calculations (21)

- 12 To compare two whole plot means at the same combination of subplot and sub-subplot treatments ( $a_0b_0c_0$  vs.  $a_1b_0c_0$ ).

$$LSD = t_{ABC} \sqrt{\frac{2[(b)(c-1)Error(c)MS + (b-1)Error(b)MS + Error(a)MS]}{rbc}}$$

and

$$t_{ABC} = \frac{(b)(c-1)Error(c)MS\left(t_{\frac{n}{2}, Err(c)df}\right) + (b-1)Error(b)MS\left(t_{\frac{n}{2}, Err(b)df}\right) + Error(a)MS\left(t_{\frac{n}{2}, Err(a)df}\right)}{(b)(c-1)Error(c)MS + (b-1)Error(b)MS + Error(a)MS}$$

∴

$$t_{ABC} = \frac{((2 * (3 - 1) * 4.681) * 2.030) + (2 - 1) * 8.705 * 2.262 + 18.626 * 2.447}{(2 * (3 - 1) * 4.681) + (2 - 1) * 8.705 + 18.626}$$

$$= 2.242$$

and

$$LSD = 2.242 \sqrt{\frac{2[(2 * (3 - 1) * 4.681) + (2 - 1) * 8.705 + 18.626]}{4 * 2 * 3}}$$

$$= 4.39$$

## Bibliography

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- Gomez, K. & Gomez, A. (1984). *Statistical Procedures for Agricultural Research*. Singapore: John Wiley & Sons, Inc.



Σας ευχαριστώ για την προσοχή σας