Multiple Latin Squares Analysis

ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ

ΓΕΩΠΟΝΙΚΗ ΣΧΟΛΗ Α.Π.Θ.

Επιστημονική Επιμέλεια:

Δρ. Γεώργιος Μενεξές

Τομέας Φυτών Μεγάλης Καλλιέργειας και Οικολογίας

Εργαστήριο Γεωργίας

ANOVA Table

Example

Two 4x4 Latin squares.

	SOV	Df
	Squares	sq – 1 = 1
*	**Row(square)**	sq(r-1) = 6
*	**Column(square)**	sq(r-1) = 6
	Treatment	r-1 = 3
	Square x Treatment	(sq-1)(r-1) = 3
*	**Error**	sq(r-1)(r-2) = 12
	Total	$sqr^2 - 1 = 31$

*Additive across squares.

Where sq = number of squares.

Example

Three 3x3 Latin squares

Square 1

				$\sum R$	
	41 (B)	25 (C)	15 (A)	81	SS Row_1 = 126.89
	20 (A)	32 (B)	24 (C)	76	SS $Column_1$ = 89.55
	22 (C)	12 (A)	21 (B)	55	SS $Treatment_1$ = 368.22
$\sum C$	83	69	60	212	SS $Error_1$ = 21.56

Square 2

				$\sum R$	
	27 (C)	28 (B)	3 (A)	58	SS Row_2 = 130.89
	4 (A)	17 (C)	9 (B)	30	SS $Column_2$ = 110.22
	22 (B)	4 (A)	17 (C)	43	SS $Treatment_2$ = 534.22
$\sum C$	53	49	29	131	SS $Error_2$ = 14.89

Square 3

				$\sum R$	
	43 (B)	27 (C)	17 (A)	87	SS Row_3 = 126.89
	22 (A)	34 (B)	26 (C)	82	SS $Column_3$ = 89.55
	24 (C)	14 (A)	23 (B)	61	SS $Treatment_3$ = 368.22
$\sum C$	89	75	66	230	SS $Error_3$ = 21.56

Calculations (1)

Step 1. Test the homogeneity of the Error MS from each square using Bartlett's Chi-square test.

Step 1.1 Calculate the Error SS for each square.

Step 1.2 Calculate the Error MS for each square.

Calculations (2)

Step 1.3 Calculate the Log of each Error MS

Square	Error SS	Error df	Error MS	Log Error MS
1	21.56	2	10.78	1.0326
2	14.89	2	7.45	0.8722
3	21.56	2	10.78	1.0326
			$\sum s_i^2 = 29.01$	$\sum \log s_i^2 = 2.9374$

Step 1.4 Calculate the Pooled Error MS (s_p^2)

$$s_p^2 = \frac{\sum s_i^2}{\# sq} = \frac{29.01}{3} = 9.67$$

Calculations (3)

Step 1.5 Calculate Bartlett's χ^2

$$\chi^2 = \frac{2.3026(Errordf)\left[\left(sq \log s_p^2\right) - \sum \log s_i^2\right]}{1 + \left[\frac{(sq+1)}{3 * sq * Errordf}\right]}$$

Where Error df = df for one square.

$$\chi^2 = \frac{2.3026(2)\left[(3 \log 9.67) - 2.9374\right]}{1 + \left[\frac{(3+1)}{3*3*2}\right]}$$

$$= \frac{0.0869}{1.2222}$$

$$= 0.0711$$

Calculations (4)

Step 1.6 Look up the Table χ^2-value at the $\boxed{99.5\%}$ level of confidence and df = #sq-1.

$$\chi^2_{0.005;2df} = 10.6$$

Step 1.7 Make conclusions

Since $\chi^2_{calc} < \chi^2_{table}$ we fail to reject $H_o: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$ at the 99.5% level of confidence; thus, we can do the combined analysis across squares

Calculations (5)

Step 2. Calculate Treatment Totals for each square.

Treatment	Square 1	Square 2	Square 3	$\sum TRT$
A	47	11	53	111
B	94	59	100	253
C	71	61	77	209
$\sum Square$	212	131	230	573

Step 3. Calculate the Correction Factor (CF).

$$CF = \frac{Y_{...}^2}{sq * r^2}$$

$$= \frac{573^2}{3 * 3^2}$$

$$= 12{,}160.333$$

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Calculations (6)

Step 4. Calculate the Total SS

$$TotalSS = \left(41^2 + 25^2 + 15^2 + \ldots + 23^2\right) - CF$$
$$= 2{,}620.67$$

Step 5. Calculate the Square SS

$$SquareSS = \frac{\sum Sq^2}{r^2} - CF$$

$$= \frac{\left(212^2 + 131^2 + 230^2\right)}{3^2} - CF$$

$$= 618.0$$

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Calculations (7)

Step 6. Calculate the Row(Square) SS (Additive across squares)

Row(Square) SS = Row_1 SS + Row_2 SS + Row_3 SS

= 384.67

Calculations

Step 2. Calculate Treatment Totals for each square.

Treatment	Square 1	Square 2	Square 3	$\sum TRT$
A	47	11	53	111
B	94	59	100	253
C	71	61	77	209
$\sum Square$	212	131	230	573

Step 3. Calculate the Correction Factor (CF).

$$CF = \frac{Y_{...}^2}{sq * r^2}$$

$$= \frac{573^2}{3 * 3^2}$$

$$= 12{,}160.333$$

Calculations (8)

Step 7. Calculate the Column(Square) SS (Additive across squares)

Column(Square) SS = Column_1 SS + Column_2 SS + Column_3 SS

= 289.32

Step 8. Calculate the Treatment SS

$$TrtSS = \frac{\sum TRT_i^2}{sq * r} - CF$$

$$= \frac{(111^2 + 253^2 + 209^2)}{3 * 3} - CF$$

$$= 1{,}174.22$$

Calculations (9)

Step 9. Calculate the Square X Treatment SS.

$$Sq * TrtSS = \frac{\sum (SqXTrt)^2}{r} - CF - SquareSS - TrtSS$$

$$= \frac{(47^2 + 94^2 + 71^2 + ... + 77^2)}{3} - CF - SquareSS - TrtSS$$

$$= 96.45$$

Step 10. Calculate Error SS (Additive across squares)

Error SS = Error_1 SS + Error_2 SS + Error_3 SS

Error SS = 58.01

Calculations (10)

Step 11. Complete the ANOVA Table.

SOV	Df	SS	MS	F (Squares and Trt are Fixed effects)
Square	Sq-1 = 2	618.0		Non-valid F-test
Row(Sq)	Sq(r-1) = 6	384.67		Non-valid F-test
Column(Sq)	Sq(r-1) = 6	289.32		Non-valid F-test
Trt	r-1 = 2	1174.22	587.11	Trt MS/Error MS = 60.73^{**}
Sq X Trt	(sq-1)(r-1) = 4	96.45	24.11	Sq X Trt MS/Error MS = 2.49^{ns}
Error	Sq(r-1)(r-2) = 6	58.01	9.67	
Total	Sqr^2-1 = 26	2620.67		

Conclusions

Conclusions:

1. The non-significant Square X Treatment interaction indicates that treatments responded similarly in all squares.

Table 1. Mean for the square x treatment interaction.

	Treatment		
Square	A	B	C
1	15.7	31.3	23.7
2	3.7	19.7	20.3
3	17.7	33.3	25.7
LSD(0.05)	--------------------ns--------------------		

2. The significant F-test for Treatment indicates that averaged across all squares, there were differences between treatments.

Calculations (11)

3.

Table 2. Mean for the treatment main effect averaged Across squares.

Treatment	Mean
A	12.3
B	28.1
C	23.2
LSD(0.05)	3.6

Step 12. Calculate LSD's

Calculations (12)

Square X Trt: Normally, you would not calculate this LSD because the F-test for the interaction was non-significant. However, if it would have been significant, you would have calculated the LSD using the following method:

$$LSD_{SqXTrt} = t_{a/2;errordf}\sqrt{\frac{2ErrorMS}{r}}$$

$$= 2.447\sqrt{\frac{2(9.67)}{3}}$$

$$= 6.2$$

This LSD would be used for comparisons only in Table 1.

Calculations (13)

Treatment:

$$LSD_{Trt} = t_{a/2;errordf}\sqrt{\frac{2ErrorMS}{sq*r}}$$

$$= 2.447\sqrt{\frac{2(9.67)}{3*3}}$$

$$= 3.6$$

This LSD would only be used for comparisons in Table 2.

Bibliography

- Steel, R. & Torrie, J. (1986). *Principles and Procedures of Statistics: A Biometrical Approach*. Singapore: McGraw-Hill Book Company.
- Gomez, K. & Gomez, A. (1984). *Statistical Procedures for Agricultural Research*. Singapore: John Willey & Sons, Inc.
- Cochran, W. & Cox, G. (1953). *Experimental Designs*. New York: John Willey & Sons, Inc.
- Cox, D. R. (1958). *Planning of Experiments*. New York: John Willey & Sons, Inc.