Computer aided analysis and design of MIMO bilinear control systems

by D. N. Kleftouris*, BSc, MSc(Eng), PhD, L. P. Petrou*, Dipl(Eng), MSc(Eng), PhD, DIC, MIEEE, and G. D. Hassapis*, Dipl(Eng), MSc(Eng), PhD, MIEEE, MInstMC

In this paper a CAD procedure for the analysis and design of bilinear MIMO control systems is presented. Algorithms are developed for the solution of the bilinear system state-space equations, based on the fact that the bilinear system can be treated as a linear one at every sampling instant. Stability is examined by means of suitable signal and induced system norms. Finally, a bilinear system is analysed and designed, using the developed CAD procedure.

1 Introduction

Bilinear systems have been extensively studied in recent years for two primary reasons. First, bilinear systems are feasible mathematical models for a broad class of physical processes, e g, biological processes (Williamson, 1977). Secondly, physical systems with non-linear dynamics, which cannot be approximated accurately by linear models, sometimes can be better represented by bilinear ones, thus avoiding the complexity of non-linear system analysis (Mohler & Rink, 1969).

A thorough examination of many aspects of bilinear systems is given by Bruni *et al* (1974). Considerable work has been carried out on the design of state observers for bilinear systems (Derese *et al*, 1979; Derese & Noldus, 1980), on the canonical forms for bilinear input–output maps (Pearlman, 1978), and on the realisation theory of systems with discrete-time bilinear response maps (Denham, 1981).

The aim of this paper is to develop a CAD procedure for the analysis and design of MIMO bilinear systems. By introducing a zero-order-hold in cascade with the bilinear system, the level of inputs is maintained constant within every sampling interval. Therefore the bilinear system can be considered as a linear one at every sampling instant. Bilinear system responses are evaluated in the time-domain using input–output relations. Algorithms are based on the concept of the *k*-time sequence matrix (KTSM) (Dimirovski *et al*, 1979). The procedure allows the introduction of PID controllers in the system forward path and their tuning according to error performance criteria. Stability is examined by means of suitable signal and induced system norms and channel failure is simulated, in order to investigate system integrity. Finally a bilinear two-tank system is studied using the developed CAD procedure.

2 Formation of bilinear systems

Bilinear systems are linear with respect to state and control, but not jointly. The bilinear system description is embedded in the feedback system configuration shown in Fig 1. Their continuous and discrete descriptions are given by Eqns (1) and (2) respectively:

$$\dot{\mathbf{x}} = A\mathbf{x} + \sum_{i=1}^{n} u_i BL_i \mathbf{x} + B\mathbf{u} \qquad \ldots(1)$$

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$

$$\dot{\mathbf{x}}_{k+1} = A\mathbf{x}_k + \sum_{i=1}^{n} u_{i,k} BL_i \mathbf{x}_k + B\mathbf{u}_k$$

$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k \qquad \ldots(2)$$

where:

- $\mathbf{x}$ is 1-dimensional state vector
- $\mathbf{u}$ is n-dimensional control vector
- u_i is the ith component of u
- A, B, C, D are the state space matrices and BL_i are $n \times n$ constant matrices representing the bilinear terms.

The insertion of the bilinear terms in the state-space equations of a linear system gives a simple non-linear structure, which allows the application of analytical procedures already applied to linear systems (Bruni *et al*, 1974). Furthermore, their performance in terms of controllability and optimisation compared with that of linear systems is greatly improved (Mohler & Rink, 1969).

In addition, many real systems are of the bilinear type. For example, the use of an observable bilinear model of the microbial cell growth in waste treatment systems permits the estimation of the unobservable state variables (Williamson, 1977). In a recent work (Figalli *et al*, 1984), the application of optimum feedback control to an induction motor, represented by a bilinear model, gives a very satisfactory control law which eliminates the oscillations and reduces the overshoots.

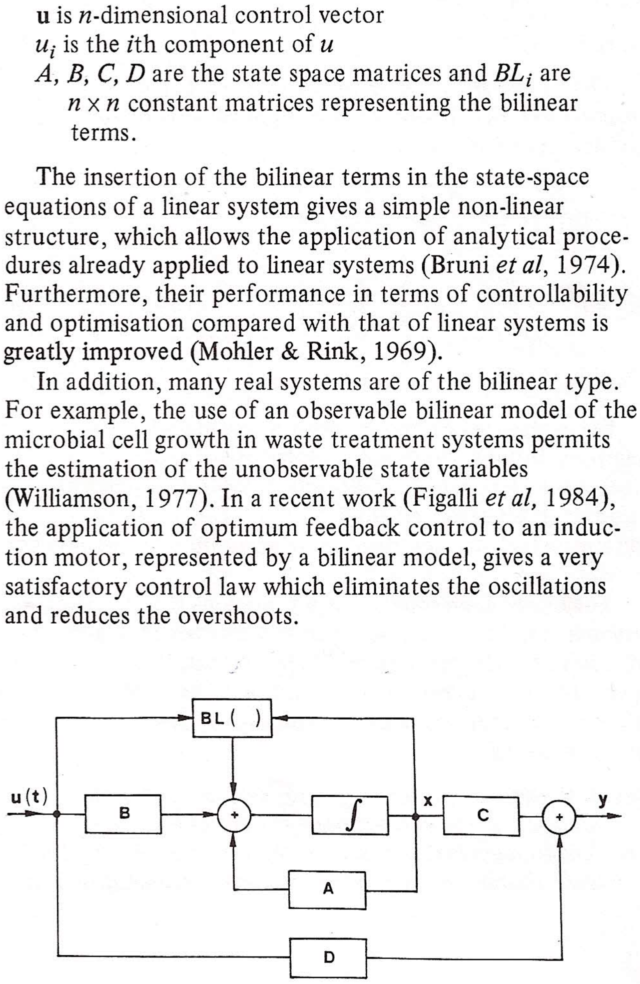

Fig 1 Bilinear system block diagram

* Department of Electrical Engineering, University of Thessaloniki, Greece.

Another advantage of the bilinear systems is that a broad class of input–output models can be reduced to the bilinear form (Brocket, 1972).

In conclusion, there is a large category of cases where the bilinear model can be employed for better system performance.

3 Bilinear state space discrete time representation and I/O relations

The computation of the bilinear control system responses by a digital computer requires the conversion of a continuous to a discrete time system. The discrete time solutions are valid at equally spaced sampling instants. Furthermore, for practical reasons, it is desirable to maintain the input level of the system constant during each sampling interval and, for this purpose, a zero-order hold device is introduced in cascade with the bilinear system.

In this respect, the bilinear system description given by Eqn (1) is simplified as follows:

$$\dot{\mathbf{x}}_k = \left(A + \sum_{i=1}^{n} u_{i,k} BL_i\right)\mathbf{x}_k + B\mathbf{u}_k$$
$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k \qquad \ldots(3)$$

where $u_{i,k}$ is the ith component of the input at the kth sampling instant. Following a similar path to the linear systems mathematical approach, one obtains the instantaneous values of the state and output vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ which satisfy the discrete state space equations:

$$\dot{\mathbf{x}}_{k+1} = F_k\mathbf{x}_k + H_k\mathbf{u}_k$$
$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k \qquad \ldots(4)$$

The values of F_k and H_k given by Eqns (5) and (6), respectively, depend on the n components of the inputs at the kth sampling instant.

$$F_k = \exp\left[\left(A + \sum_{i=1}^{n} u_{i,k} BL_i\right)T\right] \qquad \ldots(5)$$

$$H_k = \left(A + \sum_{i=1}^{n} u_{i,k} BL_i\right)^{-1}\left[\exp\left[\left(A + \sum_{i=1}^{n} u_{i,k} BL_i\right)T\right] - I\right]B \qquad \ldots(6)$$

Since matrices F_k and H_k must be evaluated at every sampling instant, the computational effort required to obtain the response of a bilinear system is considerably large. It is, however, justified for that class of physical systems which are described more accurately by the bilinear model.

The closed-loop input–output relations for the discrete bilinear system may be obtained in a manner analogous to that used for the open-loop bilinear system. By introducing a sampler in each error channel of Fig 2, the following steps are used in order to obtain the closed-loop responses for servo mode operations:

(a) The value of the error signal is evaluated in each channel, at the kth sampling instant as $e_{i,k} = r_{i,k} - y_{i,k}$.
(b) The input signal to the bilinear system is evaluated in each channel by the following convolutional product:

$$u_{i,k} = \sum_{p=1}^{n} C_{i,k+1-p}\, e_{i,p} \qquad \ldots(7)$$

where $C_{i,k+1-p}$ are the PID controller time series.

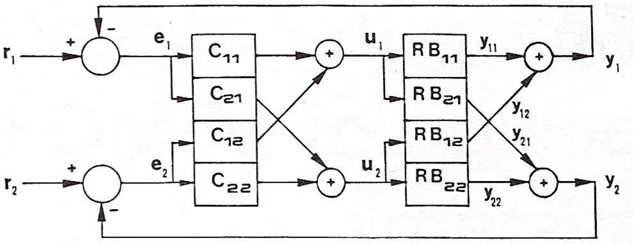

Fig 2 Bilinear closed-loop system

(c) Next, matrices F_k and H_k are evaluated, and the bilinear closed-loop responses and output signals for this particular sampling instant are computed.
(d) Steps a–c are repeated for the next (k + 1) sampling instant.

For the regulator type of MIMO bilinear control systems and for disturbances applied to the actuator channels, the above procedure is modified as follows:

in step (a), $e_{i,k} = -y_{i,k}$ instead of $e_{i,k} = r_{i,k} - y_{i,k}$, and

in step (b), $u_{i,k} = d_{i,k} + \sum_{p=1}^{n} C_{i,k+1-p}\, e_{i,p}$ instead of Eqn (7)

where $d_{i,k}$ denotes the disturbance signal in the ith input at the kth sampling instant.

A similar algorithm is used to simulate the influence of disturbances applied to the system outputs.

4 Algorithmic aspects

According to the k-time sequence notation (Dimirovski *et al*, 1979), the weighting sequence of a MIMO linear system G is defined by:

$$G = \{G\}_1^k : J_+ \rightarrow R^{n\times m} \quad k\epsilon J_+ \qquad \ldots(8)$$

with a single element

$$g(n, m) = \{g(n, m)\}_1^k : J_+ \rightarrow R^{n\times m}, \; k\epsilon J_+$$

and with the I/O mapping defined by the discrete convolution:

$$y_{m,k} = \sum_{j=1}^{n}\left(\sum_{i=1}^{k} g_{k-i}(j, m)\, u_{j,i}\right) k\epsilon\,[1, N] \qquad \ldots(9)$$

where J_+ is the set of positive integers, $R^{n\times m}$ is the set of real matrices, n is the number of inputs, m is the number of outputs and N is the number of sampling instants in the response settling time.

To assess I/O properties of the system and especially its I/O stability, norm analysis is employed to estimate subsystem and overall system gains in both closed – and open–loop structures. The analysis of a system with respect to different input signals of varying magnitudes depends on the ability to quantify input–output properties by means of suitable norms. Such signal norms and induced system norms (Desoer & Vidyasagar, 1975; Willems, 1971) can be calculated using well-known formulae, the discrete versions of which are:

$$\| \mathbf{f} \|_p = \left[\sum_{j=1}^{n}\sum_{i=1}^{N} \| \mathbf{f}_{j,i}(iT) \|^p\right]^{1/p} \qquad 1 \leqslant p < \infty \qquad \ldots(10)$$

$$\| \mathbf{f} \|_\infty = \sup \| \mathbf{f}_{j,i}(iT) \| \qquad p = \infty \qquad \ldots(11)$$

$$\| s \| = \sup \frac{\| s\mathbf{f}_{j,i}(iT) \|}{\| \mathbf{f}_{j,i}(iT) \|} \qquad \| \mathbf{f}_{j,i}(iT) \| \neq 0 \qquad \ldots(12)$$

where $\mathbf{f}_{j,i}$ is the value of the signal $\mathbf{f}$ in channel j at the ith sampling instant, while s is the gain of the operator.

As it can be seen, these norms may be regarded as generalisations of the error functionals (e g, ISE, IAE, ITAE) which are widely used to assess control system performance. However, for engineering purposes, it is sufficient to consider the norms defined by $p = 1$ (total magnitude norm), $p = 2$ (energy norm) and $p = \infty$ (least upper bound norm). Once these norms have been calculated, they can be used in conjunction with well known theorems – such as small gain, small incremental gain, bounded gain and bounded incremental gain – to estimate and quantify I/O system properties (Desoer & Vidyasagar, 1975; Willems, 1971).

Dimirovski's conversational interactive package (Dimirovski *et al*, 1979) called CSNCS (Computer Study of Non-linear Control Systems) possesses various simulation, analysis and design procedures. The package contains the necessary programs for typical memoryless and zero-order memory non-linear distortions (18 types), deterministic and stochastic signal and parameter generators, multivariable controllers and non-linear compensators, signal norms and induced norms, performance indices (ISE, IAE, ITAE) and interactive graphics.

The above-described algorithms for the simulation of bilinear systems have been added to this package. Thus the course of a typical run using the CAD facility for the analysis and design of bilinear systems, is the following:

(i) Input bilinear system state space equations and define the type of the input signal (step, impulse, sinusoidal).

(ii) The package computes and displays the open-loop response of the system for a specific sampling period. To obtain the best simulation, a number of open-loop response trials need to be carried out for various sampling periods.

(iii) Obtain closed-loop response with no controller defined. From this response, the user can get a first view of the system performance and make an initial assessment of the control requirements.

(iv) Choose a set of controller parameters and a performance criterion considered most appropriate for the case. The computerised procedure provides the new closed-loop response of the system and the value of this performance index. This step is repeated, specifying every time a new set of controller parameters until the performance index is minimised.

(v) If system integrity is required, then the breakdown of a channel is simulated by zeroing the corresponding elements of the controller matrix. The closed-loop response of the system is displayed. In case of poor performance of the system, another controller must be found by repeating step (iv).

(vi) The procedure evaluates the I/O stability of the closed-loop system by computing signal and induced system norms. The user assesses stability in the sense of these norms and either repeats step (iv) or accepts the controller in hand.

The application of this procedure is demonstrated below in the example of the two-tank system.

5 Application: a two-tank system

A two input/two output coupled-tank level control system (shown in Fig 3) is examined.

The two-tank system is an experimental set-up at Bradford University, UK (Meira, 1975). Generally, this

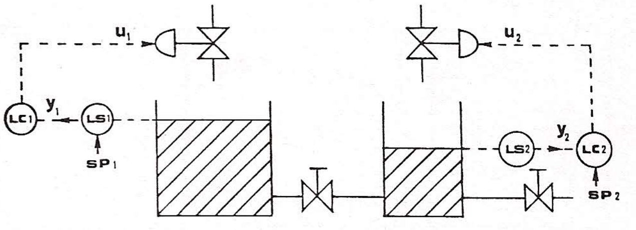

Fig 3 Two-tank block diagram

control system is widely used as an illustration of the interaction between control loops, and it provides a useful example for the multivariable control theory (Takahashi *et al*, 1972). For this system, the design of a multivariable controller is required in order to adjust the levels of the tanks to desired set-point changes, i e, servo mode operation. The regulation of the system under the influence of disturbances was not one of the control scheme objectives.

This system is described by the state-space linear Eqns (13). In these it is assumed the valves operate only over that part of their characteristics which can be regarded as linear. If, in practice, the valves move outside the limits, then significant non-linearities arise and should appear in the system model. By approximating these non-linearities by bilinear terms, the two-tank system is now described by state-space Eqns (14).

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{pmatrix}-3.88 & 3.88\\ 1.75 & -2.75\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}1.32 & 0\\ 0 & 0.582\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} \qquad \ldots(13)$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{pmatrix}-3.88 & 3.88\\ 1.75 & -2.75\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+u_1\begin{pmatrix}-0.1 & 0\\ 0 & 0.2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$$+u_2\begin{pmatrix}0.11 & 0\\ 0 & 0.05\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}1.32 & 0\\ 0 & 0.582\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} \qquad \ldots(14)$$

Control valves are usually fast compared with the process (Luyben, 1973). For this reason the time needed for a valve of the system to travel to a new position is negligible compared with the dominant time constant of the system (1.5 min). The bilinear open-loop response to a unit step reference input for a sampling period of $T = 0.2$ min is presented in Fig 4a.

Closing the loop without using any controller, the system exhibits severe interactions and significant steady-state offsets in both channels, as shown in Fig 4b. Therefore a suitable controller must be designed to improve the response of the system. In addition, this controller must preserve system stability in case of channel failure. Because of the familiarity of the PID controller and its dominant use among the control engineers, a PID algorithm has been employed.

The PID algorithm is defined by Eqn (15), where:

K_p is the proportional action gains matrix,
K_i is the integral action gains matrix,
T_i is the integral action time constants matrix,
K_D is the derivative action gains matrix,
T_D is the derivative action constants matrix and
T is the sampling period.

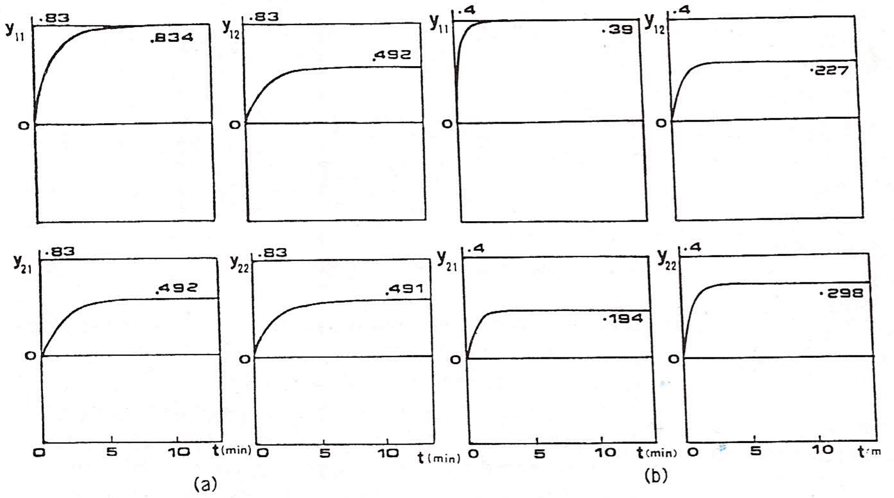

Fig 4 (a) Bilinear open-loop response. (b) Bilinear closed-loop response without controller; y_{11} is the output response in channel 1, when $u_1 = 1$, $u_2 = 0$; y_{12} is the output response in channel 1, when $u_1 = 0$, $u_2 = 1$; y_{21} is the output-response in channel 2, when $u_1 = 1$, $u_2 = 0$; y_{22} is the output response in channel 2, when $u_1 = 0$, $u_2 = 1$

$$\mathbf{u}_k = k_p \mathbf{e}_k + TK_i T_i^{-1} \sum_{j=1}^{k} \mathbf{e}_{k+1-j} + \frac{1}{T} K_D T_D (\mathbf{e}_k - \mathbf{e}_{k-1}) \qquad \ldots(15)$$

In most liquid level systems, proportional controllers are often used to give averaging level control (Luyben, 1973). Since our system possesses severe steady-state offset, the need of integral action is apparent. Since liquid level measurements are noisy, derivative action has not been used, to avoid valve position fluctuations (Shinskey, 1979). As the response of the system is overdamped and good tracking is the main control requirement, the ITAE performance criterion was considered most appropriate to tune the controller. By applying the procedure described in Section 4, the following PI controller has been derived:

$$K_p = \begin{pmatrix} 2.51 & 0 \\ 0 & 4.02 \end{pmatrix} \qquad K_i = \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}$$

$$T_i = \begin{pmatrix} 2.13 & 2.95 \\ 2.95 & 1.55 \end{pmatrix} \qquad \ldots(16)$$

Satisfactory closed-loop step responses have now been obtained, as shown in Fig 5a.

When we use the linear model (13) for the two-tank system with the PI controller (17), then the closed-loop response (Kleftouris, 1978) is as in Fig 5b. Comparing the two responses, it can be seen that the bilinear model leads to a better closed-loop response in the sense of faster rise time in the second channel and smaller interaction in both channels. Furthermore, the controller is simpler and cheaper in terms of energy requirements, because the off-diagonal elements of the proportional gain matrix are zero.

$$K_p = \begin{pmatrix} 3.82 & 3.88 \\ -2.78 & 6.24 \end{pmatrix} \qquad K_i = \begin{pmatrix} 2.94 & -2.94 \\ -2.94 & 4.73 \end{pmatrix}$$

$$T_i = \begin{pmatrix} 0.53 & 0 \\ 0 & 0.7 \end{pmatrix} \qquad \ldots(17)$$

Upon the completion of the controller design, the integrity test follows. To carry out the integrity test for the selected PI controller, the PI matrices (18) and (19) are selected for channels 1 and 2 respectively.

$$K_p = \begin{pmatrix} 0 & 0 \\ 0 & 4.02 \end{pmatrix} \qquad K_i = \begin{pmatrix} 0 & -10 \\ 0 & 10 \end{pmatrix} \qquad T_i = \begin{pmatrix} 1 & 2.95 \\ 1 & 2.95 \end{pmatrix} \qquad \ldots(18)$$

$$K_p = \begin{pmatrix} 2.51 & 0 \\ 0 & 0 \end{pmatrix} \qquad K_i = \begin{pmatrix} 10 & 0 \\ -10 & 0 \end{pmatrix} \qquad T_i = \begin{pmatrix} 2.13 & 1 \\ 2.95 & 1 \end{pmatrix} \qquad \ldots(19)$$

In Figs 6a and 6b, the closed-loop system responses for these controllers are presented. Note that, in the case of a channel failure, the other one remains stable.

Finally, to investigate the I/O stability of the closed-loop system, norm analysis is performed. Considering the

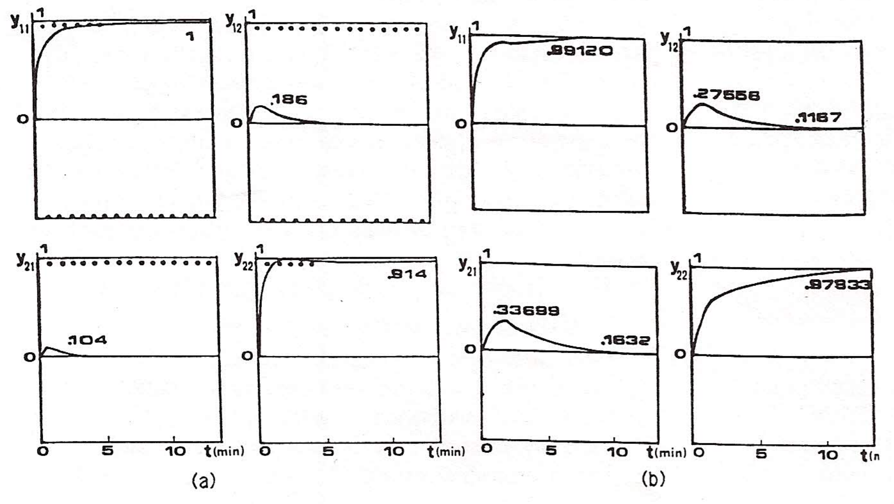

Fig 5 (a) Bilinear closed-loop response with the PI controller (16). (b) Linear closed-loop response with the PI controller (17)

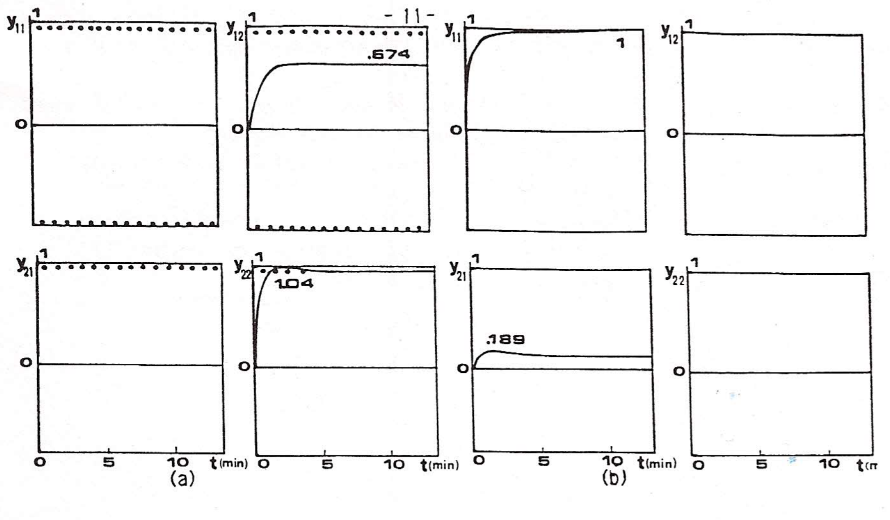

Fig 6 (a) Bilinear closed-loop response with integrity test on channel 1. (b) Bilinear closed-loop response with integrity test on channel 2

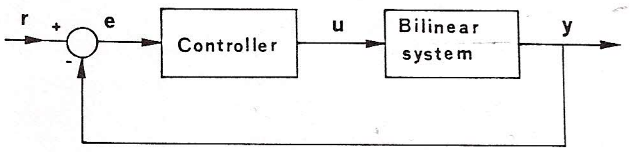

Fig 7 General feedback configuration

notation of Fig 7, and applying formulae (10) and (11) for unit step inputs over all the sampling instants ($N = 64$), the total magnitude norm ($P = 1$), the energy norm ($P = 2$) and the least upper bound norm ($P = \infty$) corresponding to reference (RPN), error (EPN), actuating (UPN) and output (YPN) signals are given in Table 1.

TABLE 1: System signal norms

	RPN	EPN	UPN	YPN
$p = 1$	128	3.79	118.56	124.94
$p = 2$	11.31	1.56	14.15	11.15
$p = \infty$	1	1	4.63	1.02

Similarly, by applying formula (12), the induced norms ($P = 1, 2, \infty$) for the controller, the bilinear system and the closed-loop system are computed and presented in Table 2.

TABLE 2: Induced system norms

	Controller	Bilinear system	Closed-loop system
$p = 1$	31.26	1.05	0.976
$p = 2$	9.045	0.787	0.985
$p = \infty$	4.63	0.221	1.02

The signal norm analysis shows that, regarding the total magnitude and energy norms, the output signal follows the input signal; i e, for reference input norms 128 and 11.31, the norms of the output signals are 124.94 and 11.15 respectively. The induced P norm analysis shows that the closed-loop operator gain is smaller than unity and, in fact, is a contracting almost identity operator. Therefore the system is finite gain I/O stable in the sense of both signal and induced norms. In the sense of the upper bound norms, no conclusions can be made because the system operator does not cause a contracting mapping.

6 Conclusions

In this paper, a systematic computer procedure has been developed for the analysis and design of MIMO bilinear systems. Well known algorithms applied to linear systems have been extended for the bilinear case. By computing system signals and operators at every sampling instant, open-loop and closed-loop responses are obtained. The analysis and design of bilinear control systems takes more computing time than the corresponding one for linear control systems. However, in the last Section, we showed that the bilinear representation of a two-tank system resulted in better responses than the linear model of the same system. Therefore, in cases where the non-linearities inherent in a system can be approximated by bilinear terms, or in cases where the physical systems are described by bilinear models, the proposed procedure can be beneficially applied.

This procedure can be easily modified to encompass the analysis and design of time-varying bilinear systems. Finally, since the internal workings of the computer programs require time series, the procedure is appropriate for designing process control schemes whose data is in the form of normal operating records.

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Research, Development and Application Notes

A shorter form of Transaction Paper

For various reasons, but particularly due to pressure of work in industry, publication of an important advance in the application of measurement and control techniques is delayed or even prevented altogether. This means that wider application may be delayed; it also denies to the author recognition of his work outside his immediate environment. Although, in industry, there is not the same pressure to publish – in fact one might say the pressure is often in the opposite direction – it must be for the general good that as much as possible is published widely.

Bearing this in mind, the Institute is prepared to publish in its Transactions what are described as 'Research, Development and Application Notes'. These would be of, say, 2000–3000 words, and be intermediate between an abstract and a full paper. They would, of course, be refereed and the work described would have to be normally publishable in the *Transactions* rather than in the journal *Measurement and Control*, but the degree of detail would necessarily be less. The objective, the methods used and the degree of success would have to be described, and perhaps wider implications, the need for the work, etc, outlined.

In addition to completed work, these notes could be used to describe major landmarks in a long programme of research or development. They could also be used to describe minor developments of research already reported in the *Transactions* or elsewhere.

It is bound to be beneficial to the author to have his work brought quickly to the attention of his profession. It is equally important for the technical excellence of any industrial R&D organisation to be recognised outside the company within which it operates. Cross fertilisation between R&D Groups and individuals is a vital requirement for rapid advances in technology. We hope readers will take advantage of this facility.