

Computer-based measurement of wastewater BOD

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Abstract. This paper describes the development of a computer-based instrument for the fast measurement of the biological oxygen demand (BOD) of wastewater. The conventional measurement method is based on laboratory analysis results that are available after a delay of five days. The BOD is assessed by the instrument described here by monitoring the change in the dilution ratio of fresh water to wastewater that has to be made in order to keep at a constant value the oxygen consumed in a special whirl bed reactor in which microorganisms are grown. Depending on the reactor volume, the algorithm for the computer control of the dilution ratio and the architecture of the computer selected, a measurement time in the range of 2 to 10 minutes can be achieved. The results of practical experience gained from building and testing such a measuring instrument are presented and discussed.

Keywords. Intelligent instrumentation, process control, water quality measurement.

1. Introduction

Reliable and accurate measurements of the influent and effluent substrate concentrations of biological oxygen demand (BOD) are required for the practical implementation of automatic control strategies for wastewater treatment plants. However, devices that can perform this type of measurement within the response time limits imposed by automatic control requirements are not generally available. A laboratory apparatus that has been reported [1] is able to measure the substrate concentration of BOD in sewage in approximately four hours, whereas the only commercially available unit [2] claims to have a measurement delay of three minutes. The detailed design of these devices is not generally published and only the basic principles on which their design is based is explained in the literature. They use a bed-type reactor in which the inner surface of a number of small plastic

Elsevier
Industrial Metrology 2 (1991) 71-84

rings serves as the growing area for microorganisms. An electronic controller tries to hold at a constant value the difference of the dissolved oxygen concentrations between the influent and the effluent streams of the reactor by adjusting the dilution ratio of the wastewater to the freshwater in the influent stream. From the monitored variations of this dilution ratio and the influent temperature the BOD of the wastewater can be inferred. It seems that the problem of designing a fast BOD measuring device is a problem of control.

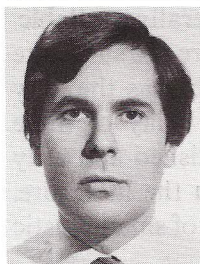
The aim of this work was to find and provide in the open literature a computer control strategy that could be used to build custom-made BOD measuring devices with measurement times comparable to those claimed in commercial products. According to this strategy a change in the wastewater BOD is viewed as a disturbance to the dilution ratio. A linear quadratic control algorithm with on-line process identification and parameter estimation of the control algorithm is applied to regulate the dissolved oxygen difference. Its operation in practice was tested by building and operating an experimental BOD measuring device.

2. Controller design

It has been proved [3] that the dynamic behavior of any reactor in which microorganisms are grown, is described by the following differential equation:

$$\frac{T_M}{k_0 c_{BA} c_w(t)} \frac{d^2 y(t)}{dt^2} + \frac{1 + T_M + T_M k_s c_{BA}}{k_0 c_{BA} c_w(t) T_M} \frac{dy(t)}{dt} + \frac{1 + T_M k_s c_{BA}}{c_w(t) T_M k_0 c_w(t)} y(t) = \frac{1}{1 + n(t)}, \quad (1)$$

where $y(t)$ is the difference of the dissolved effluent oxygen from the influent oxygen, expressed in mg/l, T_m is the mean residence time defined as the ratio of the volume V of the reactor over the constant flow Q of the influent stream, expressed in minutes, k_s is a constant characterizing the velocity of the biochemical reaction, c_{BA} is the concentration of the effluent biomass after the growth phase of the microorganisms is completed, expressed in mg/l, k_0 is a constant used in the oxygen



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mass balance equation, $c_w(t)$ is the BOD concentration in mg/l in the wastewater, and $n(t)$ is the dilution ratio defined as the ratio of the freshwater flow rate to the wastewater flow rate.

In a closed-feedback loop a controller of this system will drive the difference $y(t)$ to a desired value y_d by manipulating the dilution ratio $n(t)$ to offset the influence of the disturbance $c_w(t)$. Therefore, the dilution ratio is a measure of the degree of wastewater pollution. A mathematical relationship that can express $c_w(t)$ as a function of $n(t)$ can be derived from eqn. (1) if we assume that at steady state the oxygen difference is equal to y_d . This relationship has the following form:

$$c_w(t) = \frac{1 + T_M k_s c_{BA}}{T_M k_0 c_{BA}} y_d [1 + n(t)] = k_c [1 + n(t)], \quad (2)$$

where k_c is a calibration constant depending on the reactor characteristics.

The fact that the dynamic behavior of the reactor is expressed by a time-varying linear differential equation makes most of the established design methods of control systems not applicable. By observing, however, the operating characteristics of the physical process we note that if a computer-based controller is utilized, then it would be possible to select a sampling period significantly shorter than the period of the $c_w(t)$ changes. This would allow us to consider the differential equation that describes the reactor dynamics as a time invariant over a time equal to the period of the variations in $c_w(t)$ and study, by using computer simulation, the behavior of the system under various possible designs of controller, which are applicable to time-invariant systems. From this study the type of control algorithm and the design method that is most appropriate for the type of dynamics the reactor presents may be selected. This selection can then form the basis for the implementation of the algorithm, carrying out process parameter identification and controller design at every sampling instance or at a multiple of the sampling instances. In the following the computer-simulated results for various possible control algorithms and the design methods used are presented and evaluated. In these simulations a step-type change in c_w is translated to a step-type disturbance in $n(t)$ by using (2), and the time response of the closed-loop system is found. The derivation of the time-invariant equation, used in this simulation, was based on previous practical experience with small reactors [3]. This experience indicates that a unit, to which a constant flow rate $Q = 1$ l/min is supplied and with a mean residence time of $T_M = 5$ min, can build, in 3–5 days, and sustain a population of microorganisms if a constant and continuous consumption of $y_d = 3$ mg/l of dissolved oxygen is secured and the construction of the reactor is such that the quantity $y_d/T_M k_0 c_{BA}$ is around 14 mg/l. In order to model the behavior of this reactor mathematically one needs to know in addition to the previously mentioned construction parameters the value of its characteristic constant k_c , defined in eqn. (2). Usually this constant is determined experimentally from (2) by manually adjusting the $n(t)$ ratio for a specific BOD value of the influent wastewater to obtain a steady state $y(t)$ value of y_d . Indicative $n(t)$ and k_c values, found when $c_w(t) = 270$ mg/l, are $n(t) = 48$ and $k_c = 5.51$ mg/l. Therefore, if one uses these values in (1) the following equation describing the dynamic behavior of a specific reactor to a

specific sample of the influent wastewater is obtained:

$$0.08642 \frac{d^2 y(t)}{dt^2} + 0.093235 \frac{dy(t)}{dt} + 0.006812 y(t) = \frac{1}{1 + n(t)}. \quad (3)$$

2.1. Regulation based on pole placement by state feedback

It can be shown that (3) has the following discrete state space representation:

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma u(k), \quad (4)$$

$$y(k) = C_1 \mathbf{x}(k), \quad (5)$$

where

$$\Phi = \begin{pmatrix} 0.9474 & -0.0038 \\ 0.0487 & 0.9999 \end{pmatrix}, \quad (6)$$

$$\Gamma = \begin{pmatrix} 0.0487 \\ 0.0012 \end{pmatrix}, \quad (7)$$

and

$$C = (0 \quad 0.2357), \quad (8)$$

when the sampling period is $h = 0.5$ min and a zero-order hold is assumed on the system input $u(t)$.

The purpose of the pole placement method is to arrange a feedback of the state variables so that the values of the poles of the closed-loop system assume the values prescribed by the desired transient response characteristics. Since the two state variables cannot be measured directly the use of an observer, given by the state equation,

$$\hat{\mathbf{x}}(k) = \Phi \hat{\mathbf{x}}(k-1) + \Gamma u(k-1) + \mathbf{K}[y(k-1) - C \hat{\mathbf{x}}(k-1)], \quad (9)$$

is required.

The matrix \mathbf{K} can be computed analytically [4] by

$$\mathbf{K} = \mathbf{P}(\Phi) \mathbf{W}_0^{-1} (0 \quad 1)^T, \quad (10)$$

where $\mathbf{P}(\Phi)$ is the matrix polynomial

$$\mathbf{P}(\Phi) = \Phi^2 + p_1 \Phi + p_2 \mathbf{I}. \quad (11)$$

Here p_1 and p_2 are the coefficients of the characteristic equation of the matrix Φ and \mathbf{W}_0^{-1} is the inverse of the observability matrix of (4).

Then the linear control law is given by the intuitively reasonable relationship

$$u(k) = -\mathbf{L} \hat{\mathbf{x}}(k), \quad (12)$$

where $\hat{\mathbf{x}}(k)$ is the output of the observer.

The matrix L can also be computed using the analytical expression

$$L = [0 \quad 1]W_c^{-1}P_1(\Phi), \quad (13)$$

where $P_1(\Phi)$ is a polynomial matrix of the form shown in (11) with the difference that the coefficients p_1 and p_2 are the coefficients of the characteristic polynomial of the desired closed-loop system described by the following equations:

$$x(k+1) = (\Phi - \Gamma L)x(k) + \Gamma \tilde{x}(k), \quad (14)$$

$$\tilde{x}(k+1) = (\Phi - KC)\tilde{x}(k), \quad (15)$$

where

$$\tilde{x}(k) = x(k) - \hat{x}(k). \quad (16)$$

The control action is further enhanced with an integrator of the error $y_d - y(k)$ in order to reduce the steady state errors. Since the BOD estimates of the instrument will always be given after the system has settled down, the steady-state errors will have an immediate influence on these estimates. The equation in (4) is now augmented with a new state introduced by the integrator. The new discrete space representation is

$$\begin{pmatrix} x(k+1) \\ x_3(k+1) \end{pmatrix} = \begin{pmatrix} \Phi & 0 \\ -C & 1 \end{pmatrix} \begin{pmatrix} x(k) \\ x_3(k) \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y_d. \quad (17)$$

The pole placement design, explained previously, is now applied to the space representation of the new system and leads to the following control law:

$$n(k) = [1 - u(k)]/u(k), \quad (18)$$

where

$$u(k) = -(25.22 \quad 297.75) \begin{pmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{pmatrix} + 277.69x_3(k) + 3, \quad (19)$$

where

$$\begin{pmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{pmatrix} = \begin{pmatrix} 0.9474 & 2.6316 \\ 0.0487 & 0.4867 \end{pmatrix} \begin{pmatrix} \hat{x}_1(k-1) \\ \hat{x}_2(k-1) \end{pmatrix} + \begin{pmatrix} 0.0487 \\ 0.0012 \end{pmatrix} u(k-1) \\ + \begin{pmatrix} 11.1479 \\ 2.5166 \end{pmatrix} y(k-1). \quad (20)$$

Curve a in Fig. 1 illustrates the time response of the simulated closed-loop system over a 25 minute period for a step change in c_w from 270 to 110 mg/l. A measure of the sensitivity of the closed-loop system to modeling errors may be given by the curves shown in Fig. 2. Curves b and a illustrate the responses of the

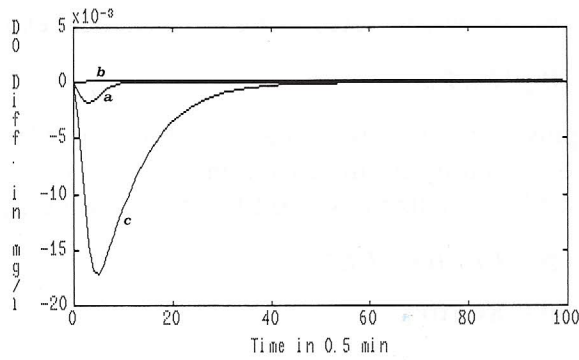


Fig. 1. Response curves of a closed-loop system to a step-type disturbance: (a), pole placement controller; (b), linear quadratic controller; (c), PID + lead compensator.

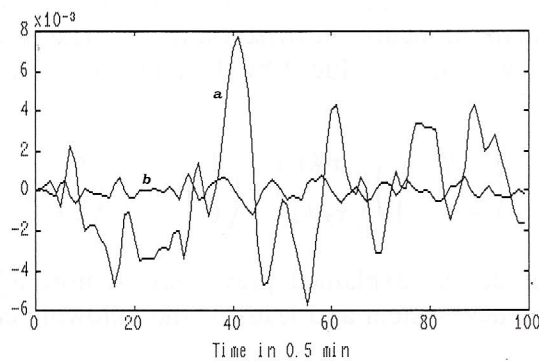


Fig. 2. Time responses of a closed-loop system with pole placement controller to: (a), controller output disturbance. (b), reactor output disturbance.

closed-loop system when random impulse disturbances following a normal distribution with mean 0.0 and variance 0.1 are applied at the reactor and controller outputs, respectively, at every sampling instant.

2.2. Linear quadratic controller

The solution of the linear quadratic (LQ) regulator problem for the discrete time system defined by (4) and (5) requires the determination of the feedback law:

$$u(k) = -L\hat{x}(k), \quad (21)$$

which minimizes the loss function:

$$J = \sum_{k=0}^{N-1} \hat{x}^T(k)Q\hat{x}(k) + Ru^2(k). \quad (22)$$

The weighting matrices Q and R have been chosen to be

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (23)$$

$$R = 1. \quad (24)$$

It is proved [4] that the matrix L can be computed from the following equation

$$L = (R + \Gamma^T S \Gamma)^{-1} \Gamma^T S \Phi, \quad (25)$$

where the matrix S is the solution of the discrete-matrix Riccati equation:

$$S = \Phi^T S \Phi - \Phi^T S \Gamma (R + \Gamma^T S \Gamma)^{-1} \Gamma S^T \Phi + Q. \quad (26)$$

The Q and R matrices selected, when compared to other matrix forms that could convert J to known performance indices [5], such as the integral square error, integral of time multiplied square error etc., lead to a closed-loop system with the fastest transient response.

The matrix equations (25) and (26) may also be used to establish the vector $\hat{x}(k)$ estimation law if the symbols L , Φ , Γ are substituted by K^T , Φ^T and C^T , and the matrices Q and R provide the covariances of the assumed superimposed noise on the system states and the output measurement, respectively. The form of these matrices that is considered is

$$Q = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix} \quad (27)$$

and

$$R = 1. \quad (28)$$

The control and state estimation laws obtained are

$$u(k) = -0.7148\hat{x}_1(k) - 0.7214\hat{x}_2(k) \quad (29)$$

$$\begin{pmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{pmatrix} = \begin{pmatrix} 0.5762 & -0.0320 \\ 0.3852 & 0.9337 \end{pmatrix} \begin{pmatrix} \hat{x}_1(k-1) \\ \hat{x}_2(k-1) \end{pmatrix} + \begin{pmatrix} 0.0487 \\ 0.0012 \end{pmatrix} u(k-1) \\ + \begin{pmatrix} 0.0101 \\ 0.1051 \end{pmatrix} y(k-1). \quad (30)$$

Curve b in Fig. 1 illustrates the transient response of the system when this control algorithm is applied. The curves in Fig. 3 illustrate the system responses to random disturbances as in Section 2.1.

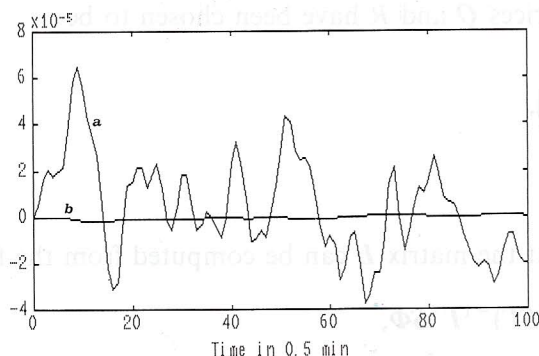


Fig. 3. Time responses of a closed-loop system with a linear quadratic controller to: (a), controller output disturbance; (b), reactor output disturbance.

2.3. PID Control

The possibility of using digital PID algorithms for this control application has also been investigated. Euler's approximations [4] for the four different structures in which PID controllers appear in industrial control systems [6], have been simulated, after being tuned by the 10% overshoot method [5] to offset the same step load disturbance used for the previous designs. The pulse transfer functions of these four different controller structures are given in the Appendix. The first function refers to a standard or otherwise non-interacting controller where the proportional, integral and derivative terms act on the error signal independently but the controller gain multiplies all three terms. The second function refers to the so-called parallel controller in which the integral and derivative terms are independent of the controller gain. In the third function the integral and derivative terms interact in the way shown in the Appendix and for this reason the corresponding controller is usually called an interacting controller. The last function refers to a non-interacting controller followed by a lead compensator, which is designed to introduce a 70° phase angle. All these functions include a filter in the derivative term.

As one can easily see in Fig. 4 the system described by (4) in the Appendix can stabilize faster when the fourth algorithm is used. The response that corresponds to this algorithm is also shown in Fig. 2 whereas the disturbance response is shown in Fig. 5.

3. Algorithm evaluation

The main objective of this work was to find a control algorithm that could drive the system considered to the dissolved-oxygen set point as fast as possible, once the BOD of the wastewater stream received has been changed. According to Fig. 2 the best way to achieve this is to implement the linear quadratic control (LQC) algorithm. This algorithm results in a 50% faster settling time and 80% less over-

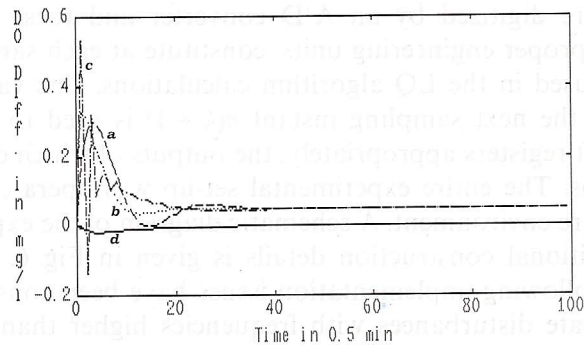


Fig. 4. Responses of a closed-loop system to a step-type disturbance; (a), non-interacting PID controller; (b), parallel PID controller; (c), interacting PID controller; (d), non-interacting PID + lead compensator.

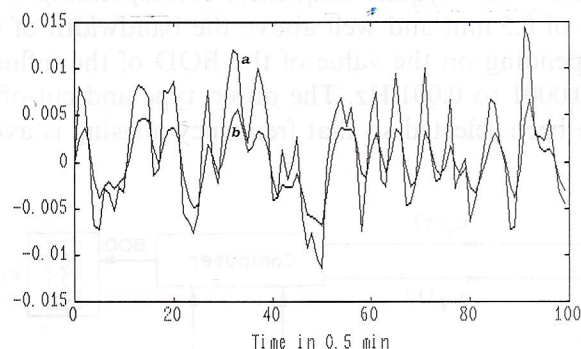


Fig. 5. Time responses of a closed-loop system with PID + compensator to: (a), controller output disturbance; (b), reactor output disturbance.

shoot when it is compared with the algorithm with the next best performance, which is the algorithm derived by applying the pole placement method. Since all the other algorithms deviate significantly from the response of the LQ algorithm any adverse influence of modeling or other errors on this algorithm and in favor of the others could not really alter the observed differences. Regarding the sensitivity of the control algorithms to modeling errors, as Fig. 3 discloses, the LQ algorithm is also the best.

4. Controller implementation

For the realization of the actions described by eqns. (25) to (30) two oxygen probes have been used to measure the incoming and outgoing reactor fluids, and two dosimetric pumps to adjust the ratio of the wastewater to the fresh water. Each oxygen probe provides, through an electronic circuit, a signal in the range of 0–5 V, corresponding to an actual oxygen concentration range of 500 mg/l, and each pump has a capacity of 1 l/min. The difference of the outputs of the two

oxygen probes are digitized by an A/D converter and these data, after being converted to the proper engineering units, constitute at each sampling instant the $y(k)$ value to be used in the LQ algorithm calculations. The value computed by the algorithm at the next sampling instant $n(k+1)$ is used to adjust the preset values of two shift registers appropriately, the outputs of which control the operation of the pumps. The entire experimental set-up was operated in a 30°C controlled-temperature environment. A schematic diagram of the experimental set-up, showing the additional construction details is given in Fig. 6. In the controller constructed the following implementation issues have been considered.

- (a) To eliminate disturbances with frequencies higher than the Nyquist frequency associated with the sampling rate a sixth-order digital low-pass Chebychev prefilter with a cut-off frequency 0.01 Hz and a sampling rate of 200 Hz has been used at the process output. The cut-off frequency is slightly below the Nyquist frequency, corresponding to the chosen sampling rate of 0.5 min and well above the bandwidth of the reactor which varies, depending on the value of the BOD of the influent stream, in the range of 0.0001 to 0.001 Hz. The order, type and cut-off frequency of the filter have been selected so that frequency aliasing is avoided and so that

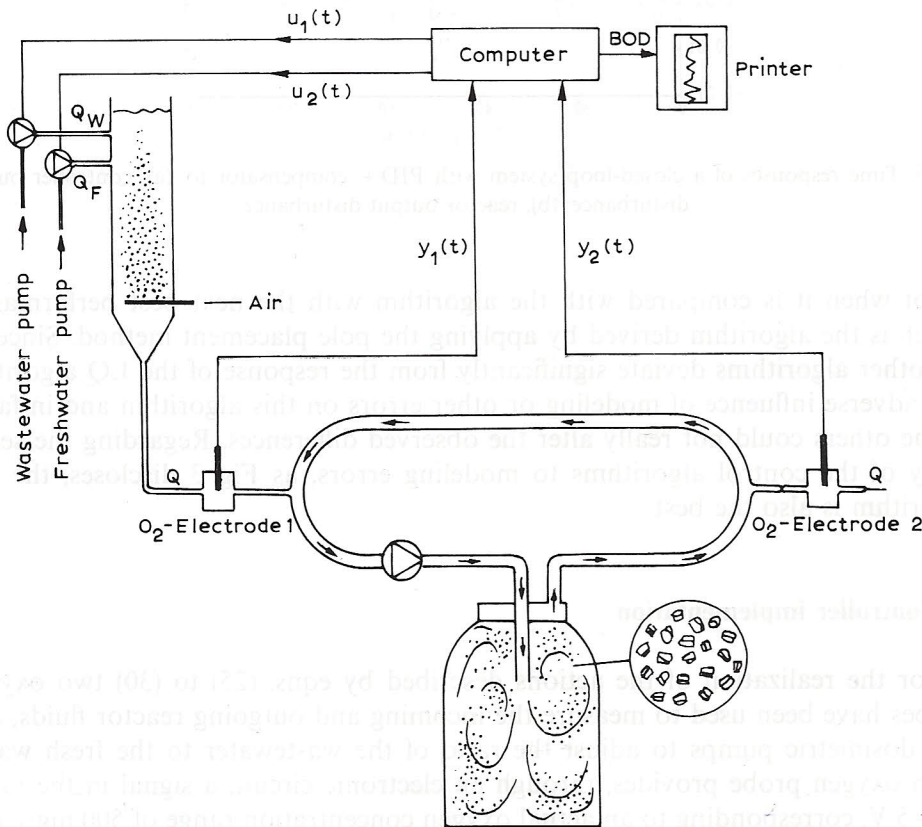


Fig. 6. Schematic diagram of experimental set-up.

the influence of filter dynamics on the response of the closed-loop system response can be neglected.

- (b) Slightly different mathematical computations were performed from those implied by (29) in order to tackle large changes of the controller output $u(k)$ that could drive the pumps to saturation, or changes less than the resolution of each pump. In fact, the applied control law is

$$u(k) = \text{sat} \{L\hat{x}(k)\}, \quad (31)$$

where the function $\text{sat } u$ is defined as

$$\text{sat } u = \begin{cases} u_{\text{low}}, & u \leq u_{\text{low}}, \\ u, & u_{\text{low}} \leq u \leq u_{\text{high}}, \\ u_{\text{high}}, & u \geq u_{\text{high}}, \end{cases} \quad (32)$$

- (c) A 12-bit A/D converter has been used to digitize the measured difference of dissolved oxygen and a double-precision floating point arithmetic has been chosen to provide a relative number accuracy of approximately 16 significant decimal digits. These selections reduced errors because of quantization in A/D and the digital representation of the various parameters as well as round-off errors in arithmetic operations.

As was explained in section 1 of the paper the control algorithm parameters should be changed from time to time to compensate for the changes incurred in the parameters of the reactor model by the BOD variations of the wastewater. A least-squares algorithm [4] executed at every other sampling instant has been used to identify the changes in the model parameters. The sequence of the last inputs $u(1), u(2), \dots, u(10)$ applied to the pumps are always recorded during closed-loop system operation along with the corresponding sequence of outputs $y(1), y(2), \dots, y(10)$. Then the matrices

$$F = \begin{pmatrix} -y(1) & u(1) \\ -y(2) & u(2) \\ \vdots & \vdots \\ -y(9) & u(9) \end{pmatrix}, \quad (33)$$

$$y = \begin{pmatrix} y(2) \\ y(3) \\ \vdots \\ y(10) \end{pmatrix}, \quad (34)$$

and

$$\Theta = (F^T F)^{-1} F^T y \quad (35)$$

are formed. The matrix Θ contains the least-squares estimate of the parameters in the following equation:

$$y(k) = \theta_1 x_1(k-1) + \theta_2 x_2(k-1) + b_1 u(k-1). \quad (36)$$

This equation is obtained if in the system eq. (4) k is replaced by $k-1$, $x_2(k)$ is expressed in terms of $x_1(k-1)$, $x_2(k-1)$ and $u(k-1)$, and the new expression for $x_2(k)$ is replaced in (5). Having found Θ and holding the last computed values for three elements of Φ and Γ , new values for the other three elements are found by solving the following system of equations:

$$\theta_1 = \phi_{11} + \phi_{12}\phi_{21}, \quad (37)$$

$$\theta_2 = \phi_{12}\phi_{22}, \quad (38)$$

$$b_1 = \phi_{12}\gamma_{12} + \gamma_{11}. \quad (39)$$

The eqns (37) to (39) have been derived during the transformations made to obtain (36). The computer then solves eqns (29) to (30) to find the new control law.

5. Experimental results

An algorithm that has been explained previously has been implemented on an IBM AT, 12 MHz personal computer, which also has a coprocessor. The computer program was developed within the MATLAB [7] environment. The measuring device was tested in the laboratory with synthetic wastewater, the BOD of which was varied over the test period as shown in Fig. 7. The measured dissolved oxygen difference and the dilution ratio $n(t)$ are illustrated in Figs. 8 and 9, respectively. The average BOD values estimated by the computer at 30°C are shown in Fig. 10. Groups of ten consecutive measurements were used to calculate the average values. These experimental results indicate that the settling time of the measuring device is surprisingly close to the simulated results but the BOD measurement may deviate from the laboratory assessment of BOD by as much as 15%.

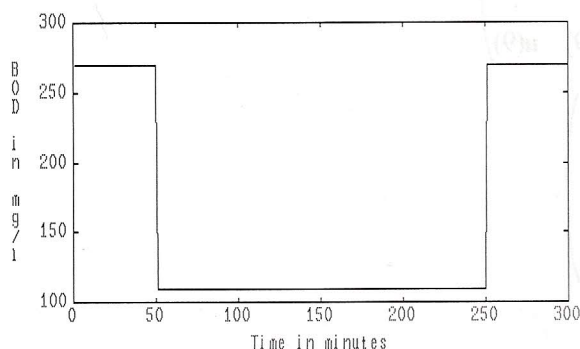


Fig. 7. BOD of synthetic wastewater input.

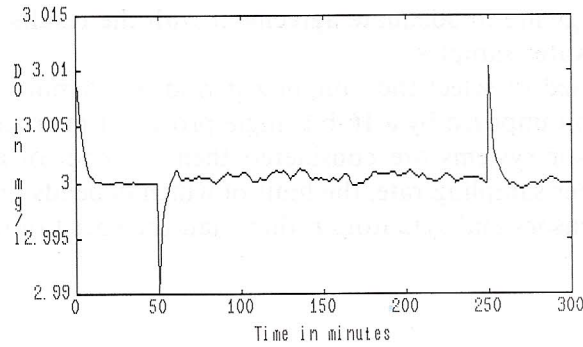


Fig. 8. Measured dissolved oxygen difference.

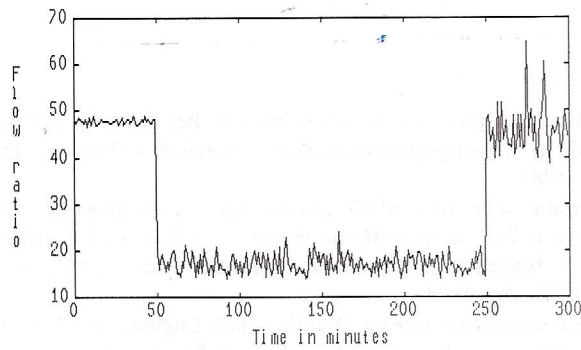


Fig. 9. Recording of the dilution ratio.

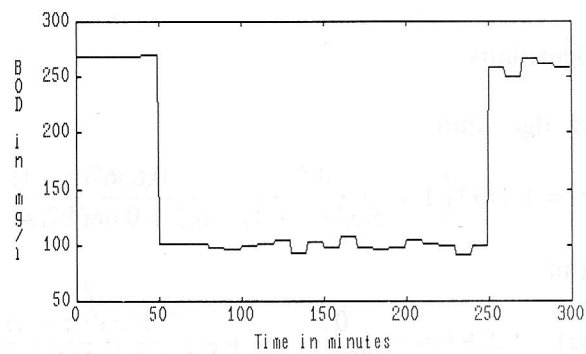


Fig. 10. Computed average BOD values.

6. Conclusions

In this paper it has been shown by computer simulation and verified experimentally that a self-tuned linear quadratic control algorithm, implemented on a small computer at a sampling period of 0.5 min, can be used to speed up drastically the response time of a BOD measuring device. The measurements obtained by using

this control strategy are in adequate agreement with the results of the laboratory analysis of wastewater samples.

The criterion used to select the sampling period of 0.5 min was the algorithm execution time limit imposed by a 16-bit single-processor microcomputer. If, however, multiprocessor systems are considered then the control algorithm can be executed at a higher sampling rate, the limit of which depends on the speed of the response of the sensors and actuators rather than the speed of the computer.

Acknowledgement

This work has been supported by grant 3293/4-3-1988 from the Greek Ministry of Industry, Research and Development Council.

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Appendix: PID algorithms

1. Non-interacting algorithm

$$H_1(z) = 1.2793 \left(1 + \frac{0.5}{5.538(z-1)} + \frac{0.6867(z-1)}{0.5 + 0.06687(z-1)} \right).$$

2. Parallel algorithm

$$H_2(z) = 1.2793 + \frac{0.5}{4.3288(z-1)} + \frac{0.85546(z-1)}{0.5 + 0.085546(z-1)}.$$

3. Interacting algorithm

$$H_3(z) = 1.1419 \left(1 + \frac{0.5}{4.432(z-1)} \right) \left(\frac{0.25 + 0.4646(z-1)}{0.25 + 0.0422(z-1) + 0.036(z-1)^2} \right).$$

4. Non-interacting PID with lead compensator algorithm

$$H_4(z) = \left(\frac{0.7079(z-1)^2 + 7520(z-1) + 0.2997}{0.0537(z-1)^2 + 2.297(z-1)} \right) \left(\frac{2.2278(z-1) + 0.8808}{(z-1) + 0.8808} \right).$$