PROCESS CONTROL WITH SAMPLED FEEDBACK

By J. O. GRAY, Z. EL-DHUWAIB and G. HASSAPIS

Department of Electrical Engineering and Electronics, UMIST, Manchester

It is required to design a controller for a chemical plant to regulate the composition of the final product in the presence of a disturbance in the input chemical composition. The product composition is monitored by a gas chromatograph so that the feedback signal is in a discrete form. A separate mass flow control loop exists which interacts with the product composition control. The design proceeds by linearising the dynamics of the plant operation about the nominal operating point and by choosing the parameters of the flow loop controller to minimise loop interaction. Various design methods are then applied to produce a range of proportional-plus-integral controllers. To minimise the settling time in the presence of a disturbance, a discrete data controller is proposed which could be realised by the use of a microcomputer. Various forms of this discrete controller are evaluated and the performance is compared with that obtained using a more conventional controller.

INTRODUCTION

The control of a particular chemical process is considered where the feedback signal is supplied from a gas chromatograph instrument which monitors the output product composition. It is required to design a conventional regulator for the process and to investigate the possible advantages of using a sampled data controller which minimises the settling time after a disturbance in the composition of the input chemical feed flow.

Due to the sampled nature of the feedback signal, conventional regulator design is not straight-forward and some form of linear continuous approximation must be employed in order to apply classical frequency domain techniques. Alternatively, design can be performed in the fictitious frequency domain using the well-known techniques of the bi-linear transform. In this paper various approximants are suggested to facilitate conventional regulator design in the real frequency domain and the resulting system performance compared with that obtained using designs based on synthesis procedures in the fictitious frequency domain. The latter were devised using a computer-aided graphical design program which greatly reduced the amount of effort normally required in such work. Finally, several sampled data compensators based on a minimum response design philosophy were devised and the resulting system performance compared with that obtained using more conventional controller forms.

DESIGN PROCEDURES

In the process under study the gas chromatograph out-puts the product composition value at discrete intervals of 30 min and, because of the nature of its operation, has an inherent measurement delay of 30 min. The resulting process feedback loop has the form shown in Figure 1 where the sampling period T is 30 min.

Various design methods have been suggested in the literature[1,2] for the derivation of linear, continuous compensators for feedback structures of this type. The most straight-forward approach is to replace the sampling and data reconstruction processes by an approximate, linear, continuous function and to employ classical design techniques. In this paper several approximation methods are suggested and evaluated. Provided that the system dynamics are amenable to a z transform analysis, an alternative and theoretically accurate design procedure is to study the behaviour of the sampled characteristic equation in the fictitious frequency domain using the artifice of the bi-linear transform and to arrive at a suitable design by again employing a classical frequency domain approach. The application of this procedure normally results in tedious algebraic manipulation and has the difficulty that there is a lack of direct correspondence between pole zero structures in the fictitious frequency and real frequency domains which can result in an undesirable structure in the final design of $H(s)$. In this work, both of these difficulties have been overcome by employing a suitable computer-aided graphical design program[3] and two different controllers have been derived in this way for evaluation.

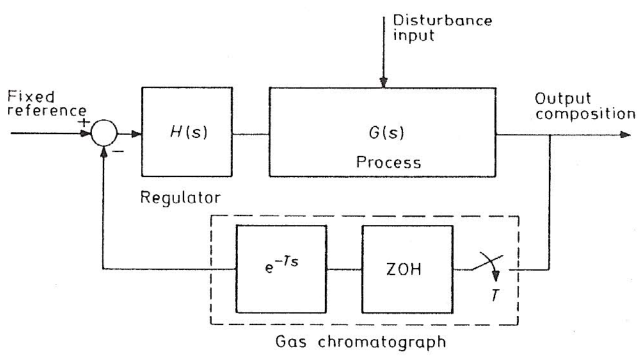

Figure 1. Compensation using a continuous regulator.

The application of special sampled data compensators to process control has been reported some years ago in the literature[4] and it is possible that the advent of relatively inexpensive micro-processor and mini-computer systems may now result in a more widespread application of such compensators. If a sampled compensator is proposed, then the feedback structure is as shown in Figure 2 where it is required to devise a suitable $D(z)$ to

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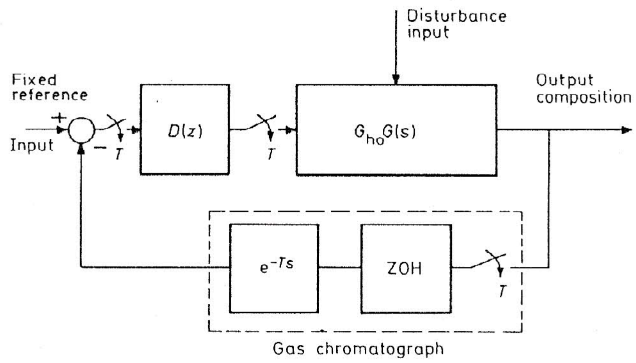


Figure 2. Digital compensation.

meet the required performance specification. A minimum time response design was specified here to minimise settling time in the composition loop to a disturbance input and two sets of compensators were derived, one of which can be used as a basis for a direct comparison with the performance of previously designed continuous regulators and the other to demonstrate how system performance can be further improved by a slight change in the plant operating procedure.

THE PROCESS

A chemical plant has an input consisting of three chemical reagents S, M and Y which must be vaporised and then reacted in a fixed proportion to form the final product. The three reagents are stored in three tanks as described below.

Tank 1 contains the reagent M only and is used to control the product composition.

Tank 2 contains all three reagents S, M and Y and is used to collect any waste mixture of these which might occur at any stage of the process. The random change of composition in this recycling tank constitutes the disturbance to the system.

Tank 3 contains the reagents S and Y only and is used simultaneously with tank 1 to control the flow rate through the process.

The schematic layout of the chemical process is shown in Figure 3 where the flow rates are shown in kilomoles per hour.

Feed flows from each tank are mixed, vaporised and passed to the reactor vessel, at the exit of which the product composition is determined by a gas chromatograph sensor. The product is separated into the desired components and proportions of these are mixed with hydrogen and recycled as shown in Figure 3. The following assumptions can be made concerning the process.

(1) The transport delays are adequately represented by pure time delays and the pots likewise by first order lags.

(2) The system is liquid filled and so liquid flows change instantaneously in response to imposed disturbances. Composition changes are delayed through transport and first order lags, the former being sufficiently small as to be safely ignored in an initial analysis.

(3) The reactor operates in the vapour phase (the first pot is a vaporiser) and the speed of reaction and speed of vapour are both infinite.

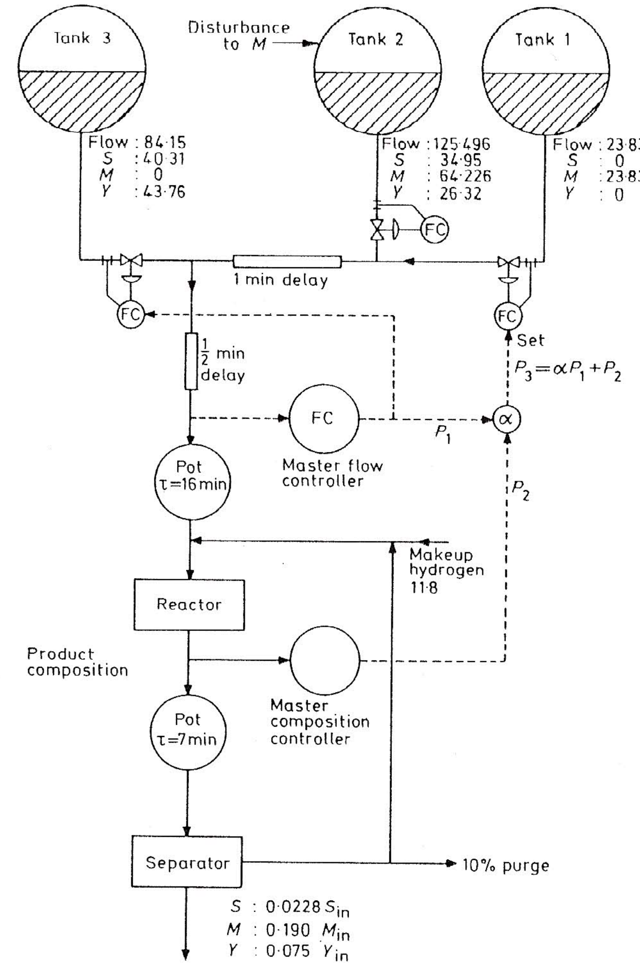


Figure 3. Process schematic.

(4) There are no lags associated with the control signals, and it can be assumed that the controllers are linear along their ranges.

(5) The reaction is an equilibrium reaction and can be described entirely by the experimentally derived equation

$$Q = 1.35M/S + 1.54Y/S - 0.98 \quad (1)$$

where M, S, Y are measured in kilomole h^{-1} at the inlet of the reactor, and Q is to be kept constant. There is no effect caused by the flow rate of the hydrogen as this is present only to suppress by-products.

(6) The chromatograph can be modelled adequately by a sample and hold, sampling once every 30 min followed by a 30 min analysis period. The desired output is not available until the end of the analysis cycle.

(7) The disturbances can be represented by subjecting the composition of tank 2 to a random disturbance every half hour of maximum amplitude 15% passed through a one hour time constant first order lag. This represents the physical event that each half hour the plant operator may or may not pour a quantity of reagent M into this tank.

(8) The separator has different efficiencies for the different reagents; the values given are linearised around the normal operating point.

The master flow controller (MFC) shown in Figure 3 sets flow rates from tanks 1 and 3 to the required value; while the master composition controller (MCC) takes the measured value of Q and sets the flow from tank 1. The value of the parameter α associated with the controller to tank 1 depends mainly on scaling of each instrument. Tank 2 is equipped with its own flow controller and the output flow from this tank can be assumed constant.

The control problem can be stated as follows. (a) To find an optimum value for the two term composition controller to tank 1 such that the disturbance response will settle in minimum time. (b) To investigate the possible advantages of using a sampled data controller.

THE TRANSFER FUNCTION

For the purposes of control two variables must be considered, the total plant throughput and the required product composition. The first variable is controlled by the master flow controller which operates on the flows from tank 1 and tank 3. It will be seen that the flow is also controlled by signals from the master composition controller and thus a degree of loop interaction exists. As previously mentioned, the flow from tank 2 is maintained constant by the use of a separate flow control loop and, because of the assumptions made in the previous section, the flow loop can be considered to possess no dynamics up to the point where it is measured by the master flow controller.

The derivation of a transfer function relating the product composition to the control signal is complicated by the non-linearity inherent in the empirical reactor equation given above and, as the expected maximum deviation of the input feed composition is relatively small, the approach adopted was to linearise the process about the normal operating point. The resulting linearised transfer function relationships are given in Figure 4 where, for convenience, the individual transfer functions have been time scaled by a factor of 60.

The relationship between the individual reagent flow rates, the control signals P_1, P_2 and the parameter α can be represented by the following equation

$$\begin{bmatrix} M_1 \\ Y_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 0.4376 \\ 0 & 0.4030 \end{bmatrix} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix} \qquad (2)$$

We can also obtain a similar equation relating product composition to P_1, P_2 and α and thus derive a combined matrix equation

$$\begin{bmatrix} \text{Composition} \\ \text{Flow} \end{bmatrix} = \begin{bmatrix} R(s) & \alpha R(s)+0.473N(s)+0.403X(s) \\ 1 & \alpha+0.437+0.403 \end{bmatrix} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix} \qquad (3)$$

This equation simply indicates the nature of the interaction between the composition control loop and the reagent flow control loop which is evident from the controller schematic arrangement shown in Figure 3.

Clearly it is desirable to reduce such interaction before considering the design of the compensator. Because of the nature of the flow loop, we can safely reduce the interactive effect in this loop of the signal from the master composition controller by increasing the loop gain and hence obtaining a high value for P_1. The resulting increase

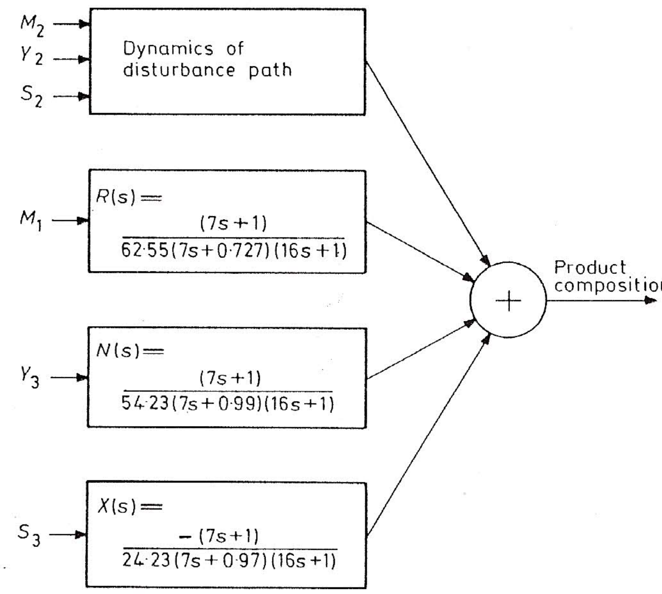


Figure 4. Transfer functions in the product composition loop.

in the interactive effect of the signal from the master flow controller in the composition loop can be countered by a correct choice of α. In this work α was chosen so that for a step change in the input flow rate we have:

$$\lim_{s\to 0} [\alpha R(s)+0.437N(s)+0.403X(s)] = 0 \qquad (4)$$

giving $\alpha = 0.41$. This criterion ensures that the interactive effect due to a step change in P_1 will be zero in the steady state. A simulation procedure revealed that the peak interaction effect in the composition control loop was now less than 2% and decayed to zero in less than 2 h. Having chosen a value of α to reduce interaction, the bias level of P_2 must now be adjusted to ensure the correct steady state operating level of the process.

CONVENTIONAL DESIGNS

In this section classical design techniques will be used in both the real and fictitious frequency domain to derive a conventional continuous regulator for the composition loop. A two element regulator structure was chosen as, due to the discontinuous nature of the feedback signal, the addition of a derivative element tended to have a de-stabilising effect. To provide a basis of comparison between the various designs, each was required to meet an arbitrarily chosen frequency domain specification of 50° of phase margin and 6 dB of gain margin.

Design in the Real Frequency Domain

The transfer function of the continuous elements of the composition loop can be expressed as

$$G_{\text{ho}}(s)\,G(s/30) = \frac{1-e^{-s}}{s} \times \frac{(s+4.286)\,e^{-s}}{33.6(s+3.116)(s+1.875)} \qquad (5)$$

where an additional scaling factor of 30 has been introduced for numerical convenience. As a first continuous system approximation, we can replace the sampler and zero order hold by a pure time delay equal to half the sampling period to obtain the open-loop transfer function

$$G_{\text{ho}}(s)\,G(s/30) \simeq \frac{(s+4.286)\,e^{-1.5s}}{33.6(s+3.116)(s+1.875)} \qquad (6)$$

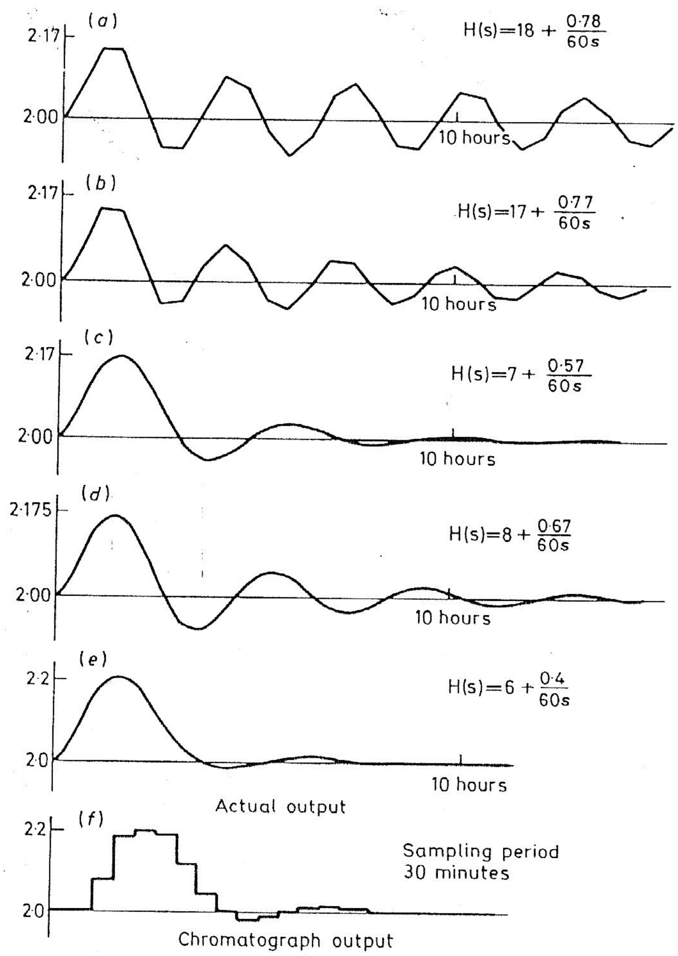


Figure 5. Analogue simulations of product composition disturbance responses with a range of proportional and integral regulators derived using various design procedures.
(a) Approximation of ZOH function by $e^{-T_s/2}$.
(b) Approximation of ZOH function by a third order rational function.
(c) Approximation of complete forward transfer function by a second order rational function.
(d) Design based on the bilinear transform procedure.
(e) Design based on the bilinear transform but using an improved performance specification.
(f) Trace on chromatograph output during the transient response given in (e).

Using classical design procedures a proportional plus integral compensator was devised for this system approximation to meet the required specification. The resultant compensator parameter values and closed loop system indicial response is shown by the analogue simulation results of Figure 5(a).

As the above approximation is not particularly accurate it is important to consider more refined approximation procedures. One approach is to use the method of moment approximants[5] to replace the zero order hold by a strictly proper rational function of suitable order. After examining second, third and fourth order approximants, it was found that no significant advantages were obtained by using an order higher than three. This yielded the transfer function approximant

$$G_{\text{ho}}(s) = \frac{1-e^{-s}}{s} \simeq \frac{s^2+3s+6}{s^3+3s^2+6s+6} \qquad (7)$$

giving

$$G_{\text{ho}}\,G(s/30) \simeq \frac{(s^2+3s+6)(s+4.286)\,e^{-s}}{33.36(s^3+3s^2+6s+6)(s+3.116)(s+1.875)} \qquad (8)$$

Using this expression as a linear continuous approximant to the open loop transfer function a two term controller was designed, the controller parameter values and resulting close loop indicial response being shown in Figure 5(b).

An alternative method of applying the method of moment approximants is to obtain an approximate proper rational function for the combined transfer functions of the sampled and continuous elements with the exception of the dead time element associated with the gas chromatograph. This results in the expression

$$G_{\text{ho}}\,G(s/30) \simeq \frac{0.214s^2-1.857s+4.286}{33.6(s+3.116)(s+1.875)}\,e^{-s} \qquad (9)$$

The corresponding compensator design results are shown in Figure 5(c).

Design in the Fictitious Frequency Domain

From a consideration of Figure 1 it is not immediately evident that a z transform analysis can be easily applied to the control loop as the error signal is not sampled. However, in this case the reference input signal is always at a constant value and we are only concerned with the relationship between the output composition value and the input disturbance signal. This allows us to manipulate Figure 1 into a standard form which is amenable to the normal analytical procedures.

The frequency response of a sampled data system can be examined by the application of the bi-linear transformation $z = (1+w)/(1-w)$ and a suitable compensator derived in the fictitious jw domain by using classical techniques such as Nichol's chart display. The compensator elements can then be isolated in either the z or s domains by the appropriate inverse transformations for realisation by either sampled or continuous elements respectively. A feature of the bi-linear transform is the lack of correspondence between pole zero structures in the w and s domains which leads to practical difficulties when, for example, it is required to produce a continuous compensator of fixed proportional-plus-integral structure. The method adopted here was to use an existing computer-aided graphical design program which accepts data in the s domain but which automatically produces displays in the w domain. Selecting a proportional-plus-integral structure in the s domain, an iterative design was undertaken with the results being displayed in a Nichol's chart format in the w domain. Two designs were produced using this method, the first giving a phase margin of 45° and a gain margin of 7 dB, and the second giving a phase margin of 70° and a gain margin of 12 dB. The Nichol's chart graphical display of the first design is shown in Figure 6 and the two designs are compared in Figure 5(d) and (e).

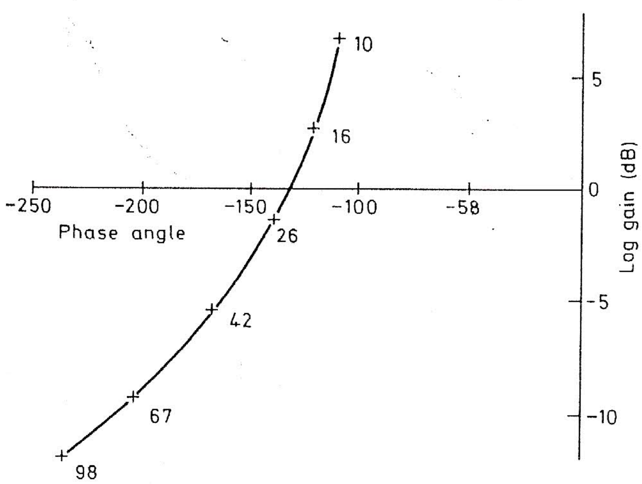


Figure 6. A computer graphical display illustrating design in the fictitious frequency domain.

ALTERNATIVE DESIGNS

The advent of inexpensive micro-computer systems offers the opportunity of investigating alternative design strategies for minimising settling time to that based on the use of proportional-plus-integral elements. Two such strategies will be considered here and used as a basis of comparison with the compensator performance given in Figure 5.

Minimal Response Design

The criteria for this design procedure are as follows. (a) The system must have zero steady state error at the sampling instants for a specified input disturbance. (b) The rise time should be equal to a minimum number of sampling periods. (c) The settling time measured at the sampling instants should be finite. (d) The compensator must be physically realisable and, for any practical application, be open loop stable. Once the form of the input disturbance signal is defined, an algebraic synthesis procedure[1,2] can be used to design a suitable compensator to meet these criteria.

A disturbance input occurs in the chemical process when a quantity of the reagent M is dumped into tank 2 which results after a one hour mixing time constant in a gradual increase in the percentage of M in the outflow from this tank with a corresponding decrease in the percentage flows of the other radicals. This change in the process input feed flow composition causes the product composition to change in the steady state according to the reactor equation previously given. As the percentage flow rates of all three reagents change simultaneously it is difficult to derive an exact analytical expression in z for the form of the input composition flow disturbance and thus, as a first approximation, it is assumed that the only change relates to a variation in the percentage of M in the flow with the corresponding changes in S and Y being ignored. With this approximation the dynamic effect of a sudden addition of a quantity of the reagent M to tank 2 on the product composition can be determined by inverting the Laplace transform expression

$$\phi(s) = \frac{K(7s+1)}{s(60s+1)(7s+0.727)(16s+1)} \tag{10}$$

where the time constants have been scaled in the same ratio as before and the constant K relates to the forward path gain and the quantity of M being added to tank 2. The corresponding z transform is

$$\phi(z) = P(z)/(1-z^{-1})(1-0.607z^{-1}) \times (1-0.0444z^{-1})(1-0.154z^{-1}) \tag{11}$$

where $P(z)$ is a polynomial in z^{-1}.

To achieve both a zero steady state error at the sampling periods and a finite settling time, the closed loop transfer function $M(z)$ must be of the form

$$M(z) = 1-(1-z^{-1})(1-0.607z^{-1}) \times (1-0.044z^{-1})(1-0.154z^{-1})F(z) \tag{12}$$

where $F(z)$ is a polynomial in z^{-1} which is determined from physical realisability considerations.

Because of the time delay associated with the gas chromatograph the open-loop sampled transfer function must be of the form

$$\phi(z) = \phi_2 z^{-2} + \phi_3 z^{-3} + \ldots \tag{13}$$

and, as $D(z)$ should not contain any time delay, we can write

$$M(z) = m_2 z^{-2} + m_3 z^{-3} + \ldots \tag{14}$$

from which, by inspection, $F(z)$ can be derived as

$$F(z) = 1 + 1.805z^{-1} \tag{15}$$

The resulting $D(z)$ is of fourth order but, because of the form chosen for $F(z)$, has an unstable pole at $z = -1.805$ and is thus globally unstable. If we choose a modified expression for $F(z)$ of the form

$$F(z) = 1 + 1.805z^{-1} + lz^{-2} \tag{16}$$

and choose the constant l such that $D(z)$ is both physically realisable and globally stable we obtain a fifth order form for $D(z)$

$$D_1(z) = \frac{(86.3z^5 - 5.57z^4 - 29.3z^3 + 5.57z^2 - 0.189z)}{(z^5 + 0.208z^4 - 1.49z^3 - 0.212z^2 + 0.486z + 0.0045)} \tag{17}$$

The minimal response design discussed above is valid only for a well-defined disturbance input and in practice the one hour time constant associated with the mixing process in tank 2 is an approximation which will vary with the operating level in the tank and the amount of extra radical added. Thus while the nature of the process input disturbance signal may always be of the form given above, the actual disturbance signal on the output concentration from tank 2 will vary and this will have an adverse effect on the settling time if a minimal response controller is applied.

If the latter signal could always be of a guaranteed form irrespective of the operating level of tank 2 or the quantity of reagent added, then the robustness of the design would be enhanced. An alternative operating procedure would be to use two tanks for the accumulation and re-use of waste products connecting one tank at a time to the process with such a connection constituting the new form of disturbance input to the process. The disturbance in the output concentration from tank 2 would then always be of the nature of a step change and because the tank mixing time constant is no longer associated with the disturbance signal path the resulting effect on product composition can be obtained by inverting the simpler transfer function

$$\phi(s) = \frac{K(7s+1)}{s(7s+0.727)(16s+1)} \tag{18}$$

where K is a constant proportional to both the forward path gain and the magnitude of the disturbance in radical concentration in the output flow from tank 2. Proceeding as before we obtain a third order open loop, stable, minimal response compensator of the form

$$D_2(z) = \frac{69.80z^3 - 13.50z^2 + 0.46z}{z^3 + 0.207z^2 + 1.20z - 0.0110} \tag{19}$$

Simulated disturbance responses using the fifth order and third order compensators are compared in Figure 7.

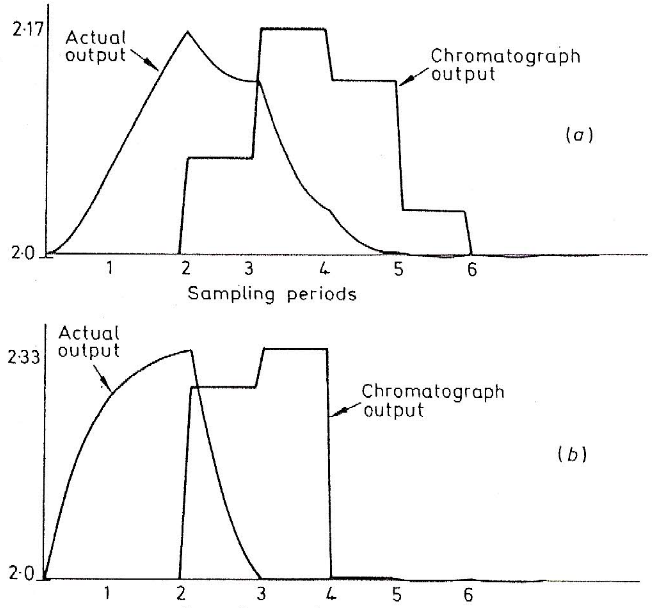


Figure 7. Digital simulations of product composition disturbances using digital compensators.
(a) Disturbance response with fifth order compensator $D_1(z)$.
(b) Disturbance response with third order compensator $D_2(z)$. Here it is assumed that the disturbance consists of a step change in the output composition of tank 2.

The corresponding forcing functions in the flow from tank 1 are shown in Figure 8. These responses have been generated using a digital simulation program on a large digital computer.

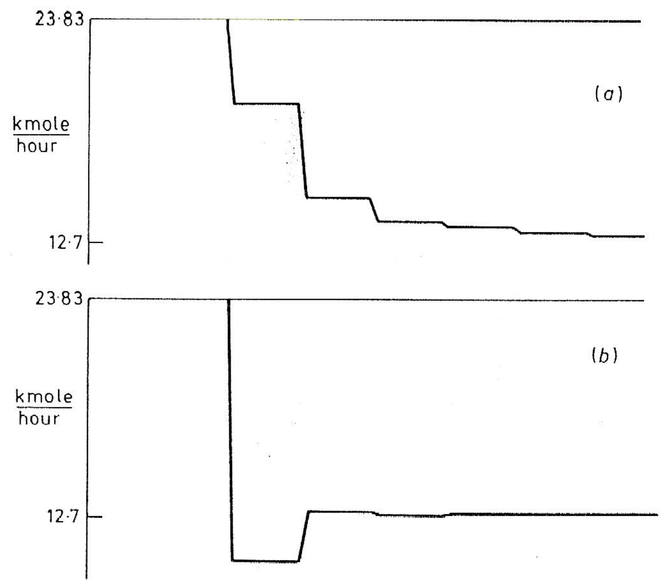


Figure 8. Forcing functions on tank 1 flow controller.
(a) With fifth order compensator $D_1(z)$.
(b) With third order compensator $D_2(z)$.

STATE SPACE DESIGN

In the minimal response design procedure outlined above, the value of one variable only, the output product composition, is controlled and no account is taken of other system variables such as the rate of change of product composition at any sampling instant. As a result such designs are prone to inter-sample ripple effects and a better result can often be achieved by using state space procedures[2] where the behaviour of a set of system variables can be specified at the sampling instants.

A linear dynamical system can be represented by the equation set

$$\left.\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{r}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned}\right\} \tag{20}$$

where $\mathbf{x}(t)$, $\mathbf{y}(t)$ and $\mathbf{r}(t)$ are state, output and input vectors respectively and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are system matrices. For any input equation set (20) has a solution of the form

$$\mathbf{x}(t) = \mathbf{\Phi}(t-t_0)\,\mathbf{x}(t_0) + \int_{t_0}^{t} \mathbf{\Phi}(t-\tau)\,[\mathbf{B}\mathbf{r}(\tau)]\,\mathrm{d}\tau \tag{21}$$

where

$$\mathbf{\Phi}(t-t_0) = e^{\mathbf{A}(t-t_0)}$$

If the system must reach a desired condition in response to a specific input we can write

$$\mathbf{x}(t_f) = \mathbf{\Phi}(t_f - t_0)\,\mathbf{x}(t_0) + \int_{t_0}^{t_f} \mathbf{\Phi}(t-\tau)\,[\mathbf{B}\mathbf{r}(\tau)]\,\mathrm{d}\tau \tag{22}$$

where t_t is the time when the response is equal to the desired value at the end of the transient period. Assuming that $t_0 = 0$ and that we require

$$x_1(t_t) = x_1 \quad x_2(t_t) = x_3(t_t) = \ldots = x_n(t_t) = 0 \quad (23)$$

the governing equation is

$$\mathbf{x}(t_t) = [x_1, 0, \ldots, 0]^{\mathrm{T}} = \mathbf{\Phi}(t_t)\,\mathbf{x}(0) + \int_0^{t_t} \mathbf{\Phi}(t-\tau)\,[\mathbf{Br}(\tau)]\,\mathrm{d}\tau \quad (24)$$

If we apply this design philosophy to the discrete data system shown in Figure 9, we obtain a discrete form of

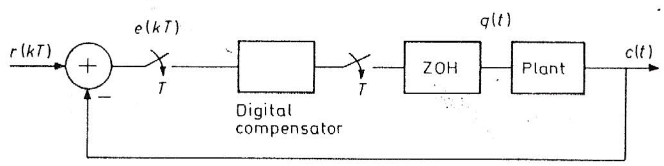


Figure 9. A control system with a digital compensator.

the state transition equation

$$\mathbf{x}[(k+1)T] = \mathbf{\Phi}(T, K_k)\,\mathbf{x}(kT) + \mathbf{F}(T)\,\mathbf{r}(kT) \quad (25)$$

where

$$\mathbf{F}(T) = \left(\int_0^T \mathrm{e}^{\mathbf{A}\tau}\,\mathrm{d}\tau\right)\mathbf{B}$$

$$K_k = K(kT) = q(kT)/e(kT)$$

$$t_t = (k+1)T, \quad t_0 = kT$$

and

$$\mathbf{r}(t) = \mathbf{r}(kT), \quad kT \leqslant t < (k+1)T$$

For a dead beat response it is required that

$$x_1(nT) = r(nT),$$
$$x_2(nT) = x_3(nT) = \ldots = x_n(nT) = 0 \quad (26)$$

The digital compensator required for any $r(t)$ can be derived by solving equation sets (25) and (26) simultaneously in order to determine K_k, $0 < k < n$, and hence the coefficients of the numerator and denominator of the required $D(z)$. Two compensators have been derived for the chemical process using the analysis outlined above. The first assumes that the input disturbance is caused by the addition of a quantity of reagent to tank 2 and is of the form

$$D_3(z) = \frac{81.42 - 3.6z^{-1}}{1 - 0.98z^{-1}} \quad (27)$$

The second assumes that the disturbance is caused by a step change in the output composition of tank 2 as outlined in the preceding section and is of the form

$$D_4(z) = \frac{56.78 - 1.63z^{-1}}{1 - 0.95z^{-1}} \quad (28)$$

USING A MICROCOMPUTER

A Development and Test Facility

The advent of inexpensive micro-computers has meant that on line computer control can now be considered for application to systems where it had been previously excluded on economic grounds. Although the actual electronic hardware is no longer costly, software development is at present not a trivial exercise and a suitable software development facility must be available for any practical application. In this work a simple development facility was designed consisting of a mini-computer with disc-based operating system, a micro-computer and analogue simulator. A block diagram of this facility is shown in Figure 10.

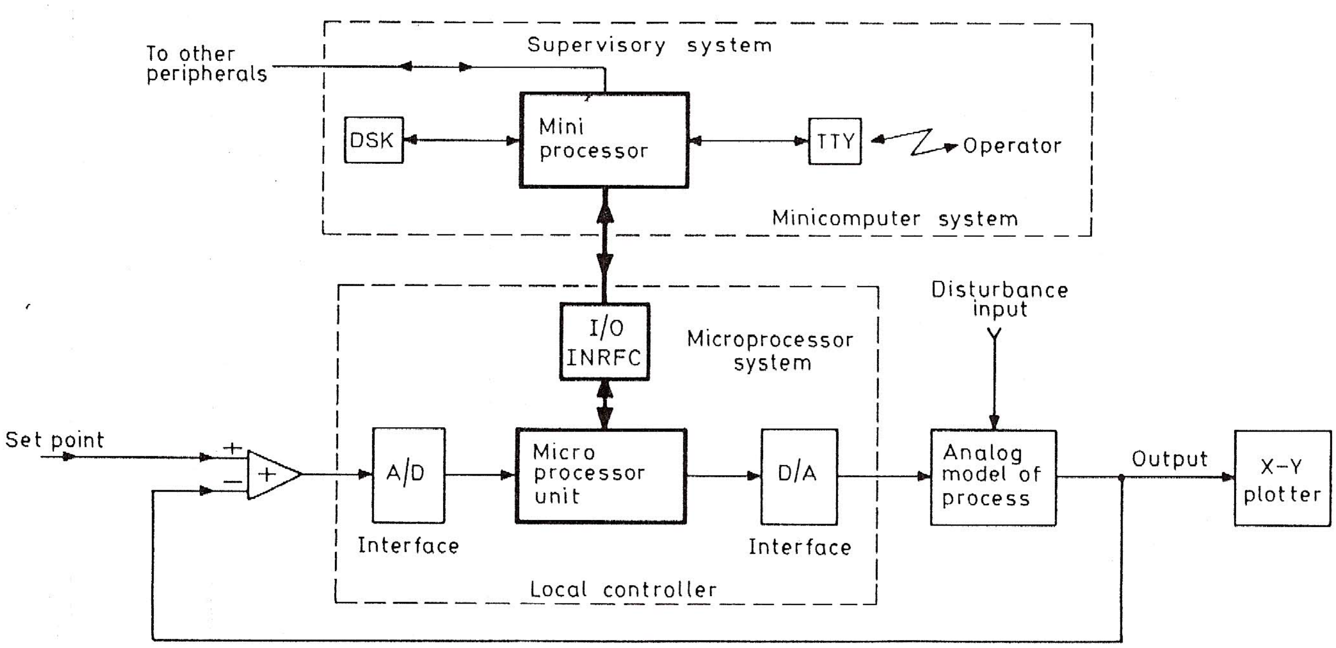


Figure 10. Configuration of a microcomputer development and test facility.

The chemical process is represented by the analogue simulator which allows for easy variations in system parameters and, if required, the injection of appropriate noise and disturbance signals at various points. The digital controller is generated by an 8 bit micro-computer which is connected *via* a 10 bit analogue/digital interface to the simulator. The mini-computer available for use was a 12 bit machine which had a resident cross assembler for the micro-computer which allowed applications programs to be edited, checked and stored on disc. A bi-directional digital interface allows these programs to be transferred as required to the micro-computer and program transfer requires only a few milliseconds to complete.

This type of mini-computer/micro-computer link up is useful not only for program development but as a test facility for checking program performance. The control programs are realised on the same hardware and use is made of the same analogue/digital interface as will be applied in the actual plant installation, thus allowing the effects of any performance degradation due to finite word length operation or any other factor to be studied.

Experimental Results

On the assumption that the input system disturbance consists of the addition of a quantity of reagent to tank 2, responses of the output product composition can be obtained by using compensators $D_1(z)$ as shown in Figure 11(a) or $D_3(z)$ as shown in Figure 11(d). In each case traces 1 and 3 relate to responses obtained when the value of the 16 min time constant in the process transfer function is varied by $+50\%$ and -50% respectively from its nominal value. As a direct comparison the response of the system with a discrete form of the controller $H(s) = 6+(0.4/60s)$ is shown in Figure 11(c).

If the plant input disturbance is assumed to consist of a step change in the reagent composition in the output flow of tank 2 then compensators $D_2(z)$ and $D_4(z)$ can be applied to give the time response in product composition shown in Figures 11(b) and (e) respectively. As before traces 1 and 3 were obtained by making a variation of $+50\%$ and -50% respectively in the value of the 16 min time constant in the system transfer function.

By using a combination of two digital controllers and assuming that the input disturbance to the process is as given in the preceding paragraph, we can obtain the set of responses shown in Figure 11(f) where traces 1, 2 and 3 correspond to values of the 16 min system time constant as previously described. During the transient phase the controller $D_4(z)$ is applied and when the response comes within $\pm 5\%$ of the final required steady state value the algorithmic constants of the control program in the micro-computer are automatically changed to generate a simple proportional-plus-integral element of the form $H(s) = 6+(0.4/60s)$. In the test facility described above this controller is transferred in from the disc of the PDP8 to replace $D_4(z)$ at the appropriate time. In practice of course the extra controller coefficients could easily be stored in the memory of the micro-computer and used as required.

DESIGN EVALUATION

The time domain performance of the various two term compensator designs was compared using a simulation procedure, and in each case the process disturbance consisted of a sudden addition of a quantity of reagent M to tank 2 of sufficient magnitude to ultimately increase the concentration of this reagent in the tank outflow by 15%. For design of the two term compensators in the frequency domain, arbitrary specifications of 50° phase margin and 6 dB gain margin were used. The first two designs were derived by simply replacing the sampling process and zero order hold by linear continuous approximations. Using the simplest approximation a fairly oscillatory response was obtained, as shown in Figure 5(a), and little improvement resulted from using an alternative third order rational function approximation based on the method of moments. When, however, the latter procedure was used to obtain a second order rational function approximation for the combined process transfer function and zero order hold, a much improved design resulted which was, in fact, superior to the design based on an iterative procedure in the fictitious frequency domain. Using the latter method and a more conservative specification of 70° phase margin and 12 dB gain margin gave the controlled performance shown in Figure 5(e) which was indistinguishable from the best performance obtained using manual adjustment on the simulation. By employing the correct form of approximation it is clearly possible to evolve a successful design in the real frequency domain without the manipulative difficulty associated with a fictitious frequency domain analysis.

In comparing Figures 5(b) and (c) it is interesting to note that the performance appears to be influenced not by the degree of truncation used in the approximation series but in the way the approximation method was applied to obtain the continuous transfer function on which the design was based. If a computer graphical design facility is available, then iterative design in the fictitious frequency domain is a simple process and has the advantage of avoiding any approximation procedures. The inherent weakness of any frequency domain procedure used in this problem is, however, the reliance which must be placed on the use of frequency domain performance functionals in order to meet a required specification in the time domain.

The application of a fifth order sampled data minimal response compensator effectively reduces the process settling time achieved by the best two term continuous compensator by a factor of two, although it will be seen from Figure 7(a) that a small ripple effect persists for a few additional sample periods. If the process operation is so modified that the third order sampled data compensator can be applied, then a further improvement in system performance is achieved with a corresponding absence of ripple phenomena. The results given in Figure 8 show that the forcing functions produced by these sampled data compensators are realistic in both magnitude and form, and no practical difficulties should arise from this aspect of their operation.

The advantages of using a minimal response design are that performance can be directly specified in the time

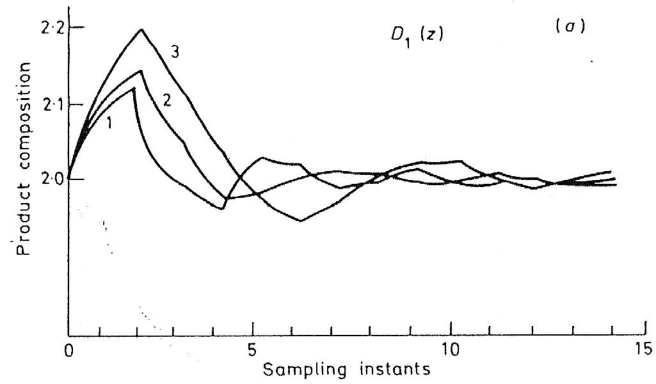
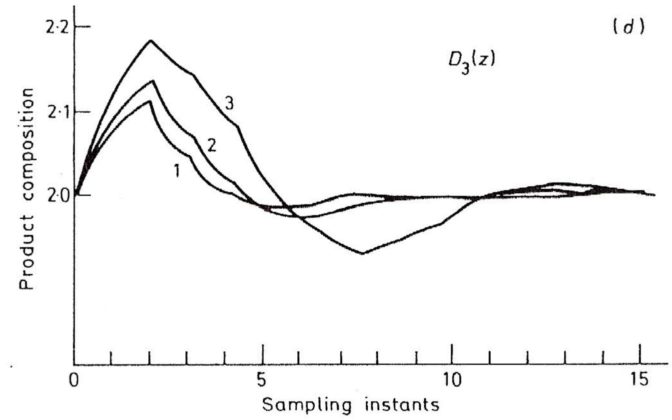
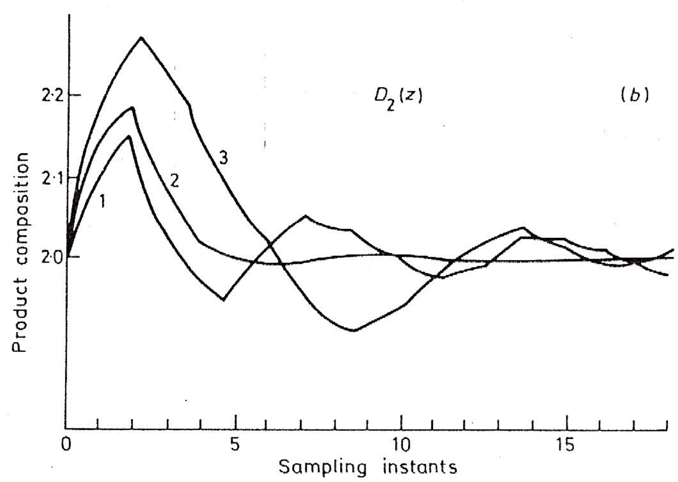


Figure 11. Hybrid simulation of product composition disturbance response obtained using the micro-computer test facility.

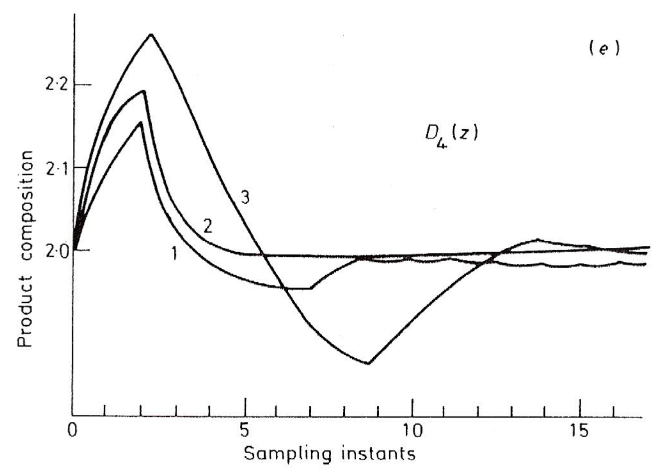
(a) With digital compensator $D_1(z)$. Trace (2) is obtained with the system 16 min time constant at its nominal value. Traces (1) and (3) relate to responses obtained when this time constant is varied by $+50\%$ and -50% respectively.



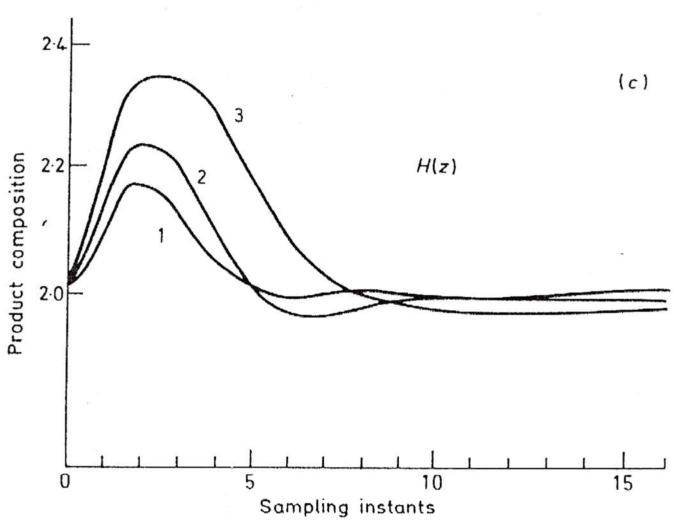
(d) Similar to (a) with compensator $D_3(z)$.



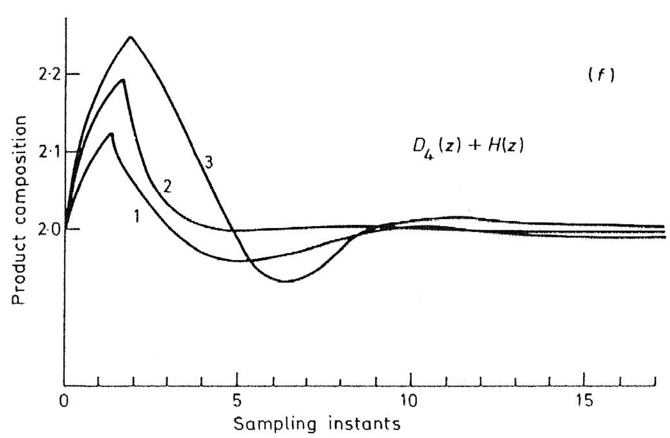
(b) Similar to (a) with compensator $D_3(z)$.



(e) Similar to (a) with compensator $D_4(z)$.



(c) Similar to (a) with a digital compensator generating a proportional plus integral element $H(z)$.



(f) Dual controller response. Similar to (e) with $D_4(z)$ being applied during the transient phase and $H(z)$ being applied when disturbance response decays to within $+5\%$ of final steady state value.

domain and the resulting controlled performance is superior to that obtained by a more conventional regulator design. Several comments must, however, be made. (a) The method depends on the cancellation of undesirable system poles and zeros and small variations in system operating parameters will affect the response. (b) The design gives a result which is correct only at the sampling instants and inter-sample ripple effects can occur. These can be theoretically avoided, however, if an alternative state space design procedure is adopted. (c) System performance is sensitive to computational round off errors and also errors due to quantisation effects. (d) The design is optimum only if the form of input signal is accurately specified.

Using the test facility described above the errors involved in any practical realisation of a minimal response design with a limited word length micro-computer can be evaluated and the effects of large system parameter variations examined. The practical performance of these controllers can also be directly compared with corresponding controllers derived using state space procedures.

A comparison of the digital simulation response of Figure 7(a) with trace 2 of Figure 11(a) clearly indicates the deterioration in controlled performance arising from errors in the analogue simulation and round off and quantisation errors due to the use of the micro-computer system. A similar result is obtained by comparing the response of Figure 7(b) with trace 2 of Figure 11(b). With the 16 min time constant set to its nominal value (trace 2 in each case) a comparison of Figures 11(a) and (d) and Figures 11(b) and (e) indicates as expected that the controllers $D_3(z)$ and $D_4(z)$ derived from state space methods do result in less inter-sample ripple than the corresponding controllers $D_1(z)$ and $D_2(z)$ obtained using the minimum response design procedures.

The two digital design methods can be further evaluated by considering three performance indices. (a) The system settling time t_s which is defined as the time for the product composition response to settle to within $\pm 2\%$ of its final steady state value after a disturbance transient. (b) An index of final product quality defined as

$$J_q = \int_{t_s}^{t_\infty} t|e(t)|\,\mathrm{d}t$$

where $e(t)$ is the instantaneous deviation of the product composition time response from the demanded steady state value and $t_\infty \gg t_s$. (c) An index of the cost of the control action defined as

$$J_c = \int_0^{t_s} |e(t)|\,\mathrm{d}t$$

Using these indices the following evaluation can be made.

(a) Assuming that the 16 min time constant is at its nominal value of 16 min (trace 2) then the controlled composition performance with $D_3(z)$ in Figure 11(d) is superior in every respect to the controlled performance with $D_1(z)$ in Figure 11(a). However, if the time constant is in error by -50% from its nominal value (trace 3) then the situation is completely reversed in favour of $D_1(z)$. If a $+50\%$ variation in the time constant is considered (trace 1) the $D_3(z)$ is again superior for all performance indices.

(b) Comparing traces 2 in Figures 11(b) and (e) shows that a better product quality, shorter settling time and significantly lower cost of control action results from using $D_4(z)$ rather than $D_2(z)$. If traces 3 are compared then $D_2(z)$ yields shorter setting time but worse final product quality than $D_4(z)$, the cost of control action as defined being almost the same for both controllers. $D_4(z)$ is again superior in both J_q and J_c if traces 1 are compared although the setting time is similar to that achievable using $D_2(z)$.

Apart from realisability considerations, the choice of digital design method in this example will thus depend on the range of variation actually expected from the time constant parameter under operating conditions. If a micro-computer is to be used for controller realisation, then the superior dual controller performance of Figure 11(f) can be achieved at negligible extra cost in programming effort or hardware complexity and represents a compromise between the requirements of short settling time and accurate ripple free steady state operation.

CONCLUSIONS

There are no special difficulties in designing a conventional two term regulator for the sampled feedback system shown in Figure 1. Compensator synthesis can be performed in the real frequency domain using a suitable continuous approximation for the sampler and zero order hold function or, alternatively, the methods of the z transform and fictitious frequency domain can be employed, preferably using a computer-aided graphical design facility. The regulator has the advantage of using conventional control elements and the overall system performance will be relatively insensitive to system parameter variations provided that the loop dynamic gain is sufficiently high.

Minimal response compensators of the form described offer the possibility of theoretically optimum system performance and they can now be realised by relatively inexpensive micro-processors. However, such compensators are sensitive to errors in the assumed values for the system parameters and also to both computational and conversion errors associated with the micro-processor and its inter-face elements. In the presence of such errors, controlled performance rapidly deteriorates from the theoretical optimum.

Apart from cost considerations, an advantage of using micro-computer systems for control is the inherent operational flexibility which can be exploited as shown here to realise a dual digital compensator in order to obtain a compromise between required transient and steady state controlled response. The micro-computer development and test facility described here proved to be useful for both micro-computer software development and system simulation and a similar facility is to be recommended in any process control study involving micro-computer application.

SYMBOLS USED

A, B, C	system matrices
D	transfer function of digital compensator
e	exponential function

$e(t)$ instantaneous deviation of product composition from set point
$\mathbf{F}(T)$ an integral function of the state transition matrix
$F(z)$ a polynomial in z^{-1}
$G(s)$ linear system transfer function
G_{ho} zero order hold element
$H(s)$ regulator transfer function
J_c, J_q controlled performance indices
j $\sqrt{(-1)}$
K system gain value
K_k gain of digital compensator at time kT
l a constant, equation (16)
M a chemical reagent
M, M_1, M_2, M_{in} flow rates of reagent M (kilomole h^{-1})
$M(z)$ closed loop digital transfer function
m_2, m_3 coefficients in equation (14)
$N(s)$ process transfer function
n number of state variables in the system
$P(z)$ a numerator polynomial in z^{-1}
P_1, P_2 control signals from the master flow controller and master composition controller respectively
P_3 control signal to adjust flow from tank 1
Q non-dimensional equilibrium factor in the chemical reactor
$q(t)$ output signal from zero order hold unit
$R(s)$ process transfer function
$r(t)$ input signal
$\mathbf{r}(t)$ input signal vector
S a chemical reagent
S_1, S_2, S_3, S_{in} flow rates of reagent S (kilomole h^{-1})
s the Laplace operator
T matrix transposition operator
T sampling period
t time
t_0 initial time
t_f time when response is equal to a desired value at the end of a transient period
t_s system settling time to $\pm 2\%$ of final steady state value
t_∞ a time greatly in excess of t_s
w fictitious complex frequency
$\mathbf{x}$ state vector
$X(s)$ process transfer function
Y a chemical reagent
Y, Y_2, Y_3, Y_{in} flow rates of reagent Y (kilomole h^{-1})
$\mathbf{y}$ output vector
z z transform operator
α a scaling factor for P_1
τ variable of integration
$\mathbf{\Phi}$ state transition matrix
$\phi(z)$ open loop sampled data transfer functions
ϕ_1, ϕ_2 coefficients in equation (13)

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ADDRESS

Any correspondence in connection with this paper should be addressed to Dr J. O. Gray, Department of Electrical Engineering and Electronics, University of Manchester Institute of Science and Technology, PO Box 88, Sackville Street, Manchester M60 1QD.

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