Proceedings of the 10th International Conference of Students of Systematic Musicology (SysMus17), London, UK, September 13-15, 2017.
Peter M. C. Harrison (Ed.).

Chord Encoding and Root-finding in Tonal and Non-Tonal Contexts: Theoretical, Computational and Cognitive Perspectives

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ABSTRACT

The concept of root is of great significance in chord encoding in tonal music. Is this notion useful in non-tonal idioms or should it be extended, changed or abandoned in different musical contexts? A series of harmonic excerpts from diverse idioms are examined through the application of different root-finding and chord encoding models, such as Parncutt's perceptual virtual pitch root-finding model, the harmonic system of Paul Hindemith, and the General Chord Type (GCT) representation. This way, the models are tested in various contexts, such as tonal, neo-tonal, whole-tone or atonal harmonies. In this process, the abstract encoding of chords in diverse tonal or non-tonal contexts is explored, employing a utilitarian notion of 'reference tone' in cases where root ambiguity is strong and cannot be resolved.

I. INTRODUCTION

In the early 20th century, the concept of tonality has been brought to its limits. New scales along with new chords have been introduced in the spectrum of composers' tools. Music theory had to catch up to such innovations, attempting to describe new scales, how new chords are formed and how they fit in the context of a musical piece. The task of describing chords and providing a rather general theory that encodes them has proven to be a difficult task.

In tonal music labelling notations include: a) figured bass (basso continuo, intervals above a given bass note), b) roman numerals (chord functions within a certain key), c) popular music or jazz notation. In atonal and non-tonal music, the concepts of pitch class sets and interval vectors are commonly employed.

In the current paper, an encoding scheme is examined, which aims to be applicable in various idioms in a universal manner. Furthermore, the encoding of chord types is reviewed focusing on the principle of root and on the intervals involved in the simultaneity. Is root useful in every case? Would it be better if it were abandoned or is there a need to be extended or changed? On which principles can one base such a chord labelling system?

II. CHORD ENCODING

A. Historical approach on the Concept of Root

Following the seminal reference to the concept of chord in Gioseffo Zarlino's (1517-90) *Le institutioni harmoniche* (1558), music theorists attempted to discover the rules that govern chords. Jean-Philippe Rameau (1683-1764), a couple centuries later, discusses major and minor triads. Some important topics include the suggestion that the origin of all harmonies via various processes are the consonant root-position triad and the dissonant chord of a triad with an added 7th; the notion of the root (*son fondamental*) as the basis to create a chord is also introduced (Lester, 2007). Rameau realised himself that the system he proposed, regardless its great strengths, had some inadequacies and ambiguities. To illustrate this better he explains that the *sixte ajouté*, subdominant chord with added 6th, can have double meaning, either as such a subdominant or as a supertonic with added 7th.

A further revolutionary approach on the theory on chords was the functional theory of Hugo Riemann (1849-1919). He tries to solve problems, like the above, by establishing the relation of chords to the three main chord functions in a scale, namely, Tonic (T), Dominant (D) and Subdominant (S).

As music got more complex, no theory was sufficient to deal with the ambiguities that emerged. A work that attempted to fulfil this role, was that of Paul Hindemith (1895-1963) in *The Craft of Musical Composition* (1937/1945). There, based on principles like the harmonic series and combination tones he proposes two series in which intervals are ordered and become important factors in finding a chord's root and categorising them in one of the six groups.

B. Chord Grouping as a Result of Categorical Perception

Auditory scene analysis suggests that grouping as well as segregating sound information, are processes of music perception (Bregman, 1990). According to Gestalt psychology, as listeners, we perceive rather complex entities as wholes instead of their constituent parts, as in the case of complex tones or even chords (Vernon, 1934).

To name such musical entities one could mention pitches, intervals, chords, durational relationships, rhythmic patterns, or even phrases and phrase groups (Deutsch, 2013). Of course such entities are not all perceived at the same perceptual level; for instance, chords and any pitch simultaneity may be seen as being significant perceptual 'primitives' already at the level of the musical surface (Cambouropoulos, 2010).

Chord labelling means abstracting from a multitude of actual pitch renderings to a sufficiently concise encoding. In order to abstract any chord type, it is necessary to take into account some general characteristics, like perceptual equivalences and similarities. The first characteristic is octave equivalence and the other is the interval equivalence. Octave equivalence refers to the strong perceptual similarity between two tones related by octaves and relates to terms such as pitch classes or tone chroma. As far as the interval equivalence is concerned, it is derived directly from octave equivalence. Pairs of inversion related intervals, also known as interval classes, have a perceptual relationship, a fact noticed by both music theorists (Piston, 1948/1987) as well as by music perception researchers (Plomp, Wagenaar, and Mimpen, 1973; Deutsch and Roll, 1974). However, an interval class may not be perceived directly, but rather as a result of a pitch class and an interval (Deutsch, 2013).

With regards to triads, major and minor chords fall into two distinct perceptual categories (Locke and Kellar, 1973); their inversions aren't considered as different chords (Hubbard and Datteri, 2001). To compare these two types, they both have a fifth, a minor and a major third, with the thirds being ordered differently within the fifth. Therefore, factors like order of intervals and a tone of reference, or root in this case, seem to play an important role in categorising chord types.

"The roots of non-tertian chords are not defined by any generally accepted theory or by the common agreement of listeners" (Kostka, 2006, p.100-101). It is straightforward to determine the intervals involved in a certain chord, but looking for a root based upon theoretical principles can be a much harder task. The most complete work on the subject comes from Paul Hindemith (1937/1945), which, however, has received criticisms from more recent researchers (Thomson, 1965, 1993; Kostka, 2006; O'Connell, 2011).

There exists an enormous number of possible pitch simultaneities that include different numbers and combinations of notes in various transpositions and inversions. Allen Forte's (1977) theory reduces chord types to up to 11 sets with cardinality 3, 29 sets with cardinality 4, 35 with 5 and so on. Such a drastic reduction is problematic for tonal music as, for instance, major and minor chords are represented by the same pc-set. On the other hand, the traditional encoding of triad-based chords is insufficient for non-tonal music.

It seems to be interesting and useful to find a method to encode all these pitch combinations that takes into account perceptual factors (e.g. octave and interval equivalence, consonance/dissonance), and at the same time adapts to many different idioms, if not all Western music idioms, in a manner that is appropriate to them. Such a encoding scheme may be used both as an analytical tool and in compositional processes. The General Chord Type (GCT) representation, explained below, aims to cover these goals.

III. THE GENERAL CHORD TYPE REPRESENTATION

It is nearly impossible to use the same tools of music analysis in different music idioms and draw significant results. A special theory has been developed for the atonal, 12-tone or serialism, with regards to 20th century music analysis, that is the set theory, including representations like pitch class sets and interval vectors (Forte, 1977). However, it is debatable if such representations work efficiently in tonal and the non-tonal idioms in between.

In order to deal with the problem of labelling any collection of pitches within a given hierarchy (e.g. key) and also functioning properly in different harmonic contexts, the General Chord Type (GCT) representation has been proposed, which will be described below (Cambouropoulos, Kaliakatsos-Papakostas, Tsougras, 2014).

A. Description of the GCT Algorithm

The GCT algorithm aims to encode any given pitch collection based on two main parameters: a binary classification of consonance and dissonance, and a scale hierarchy. For the first, a 12-value consonance/dissonance vector is introduced, where 0 means dissonance and 1 means consonance. As for the scale, it is necessary for the definitions of a tonic (or reference note) and also to know which chord notes belong to the respective scale.

For example, the regular consonance / dissonance vector for a tonal context is [1,0,0,1,1,1,0,1,1,1,0,0]; this means that the unison, minor and major third, perfect fourth, perfect fifth, minor and major sixth are considered consonant, whereas the rest dissonant, i.e. minor and major second, tritone, minor and major seventh.

Pitch hierarchy (assuming there is one) is given as a 'tonic' and its scale tones, e.g. 0 [0,2,4,5,7,9,11] for C major or 3 [0,2,3,5,6,8,9,11] for Eb octatonic (whole-step-half-step scale).

An input chord to the GCT algorithm is given as a list of MIDI numbers which is converted to pitch classes (i.e., MIDI numbers modulo 12) before being fed into the algorithm.

The basics of how this algorithm works on a given input chord is explained here:

GCT Algorithm

find all subsets of pairwise consonant tones
select maximal subsets of maximum length
for all selected maximal subsets **do**
    order the pitch classes of each maximal subset in the most compact form (chord 'base')
    add the remaining pitch classes (chord 'extensions') above the highest pitch of the chosen maximal subset (if necessary, add octave - pitches may exceed the octave range)
    the lowest tone of the chord is the 'root'
    transpose the tones of chord so that the lowest becomes 0
    find position of the 'root' in regards to the given tonal centre (pitch scale)
endfor

To illustrate this better, let's assume the chord consisting of MIDI pitch numbers 54. 62, 69 and 72 and try to convert them into a GCT representation. Let the key be C major: 0 [0,2,4,5,7,9,11] and consonance / dissonance vector as above. The pitches mod12 equal to [6,2,9,0] and are ordered from lower to higher [0,2,6,9].

We observe that the maximal consonant subset appears to be [2,6,9] (the rest with only two elements are [2,6], [2,9], [0,9] and [6,9]), and is considered the 'base' of the representation. Tone 0 is added to the right as an extension and is written as [2,6,9,12]. Comparing it with the given scale, 2 becomes the 'root' of the chord and it is rewritten as [2,[0,4,7,10]]. The specific chord is a major 7th chord on the 2nd degree, note D, i.e., the secondary dominant in C major.

B. Evaluating the GCT Representation

The GCT algorithm has been tested in a tonal context against the Kostka-Payne harmonic analysis dataset created by

David Temperley, where the automatic chord labelling was correct by 92.16%, compared to the Kostka-Payne ground truth (Kaliakatsos-Papakostas, Zacharakis, Tsougras, Cambouropoulos, 2015).

Even though the aim of GCT is to be applied in many other non-tonal music idioms, it has not been tested systematically on them yet. The difficulty of this task resides on the lack of a systematic approach to label symbolically the chords used in non-tonal music. Therefore, it is difficult to find similar ground truth for 20th century harmonic styles.

It is important to note that, the application of the GCT both as an analytical and compositional tool depends on the user's settings. The algorithm labels given simultaneities (taken from a harmonic reduction that has been manually constructed); it does not produce harmonic reduction and analyses automatically. However, the fact that one can 'learn' from data which chords comprise a specific idiom, and thus occur more often, can lead the algorithm a step further in doing a harmonic reduction itself.

IV. ROOT-FINDING IN NON-TONAL CONTEXTS

In this part, the effectiveness of GCT on finding a tone upon on which a chord is built will be evaluated, compared with Parncutt's tonal root-finding model (1997) based on Terhardt's theory of virtual pitch and the 'universal' theory of chord roots proposed by Hindemith (1937/1945). Apart from that, it will be paralleled with the abstract encodings of Forte's pc-sets and their efficiency on different contexts.

Note that in the current paper, the step of voicing in Parncutt's model is omitted, because in non-tonal contexts all the resulting values are really close to each other. If it is taken into account the weighted bass note tends to become the respective perceptual pitch. Also, the application of Krumhansl/Kessler profiles (1982) is being the same as the one in Parncutt (2007), where he examines the profiles of T_n-Types. It is agreed that since the concept of tonality doesn't fit on the examined excerpts, Krumhansl/Kessler profiles would bias mistakenly the results.

The application of the three models in a tonal context is tested on Beethoven's Sonata op.27 no.2 (Figure 1). All three models agree on the same roots. Here the standard concept of root can be observed. Note that Parncutt's model suggests two possible roots, because it isn't used in its full version. As far as set theory is concerned, it is less efficient to provide information on the chord degrees, thus their function in the key, and the different types of chords, since major and minor chords group together.

In Table 1 below, which corresponds to Figure 1, the pitch classes of the chords, their respective prime forms and GCTs are presented. GCTs appear in two forms: one with the 'standard' tonal consonance vector and one with a vector where all intervals are 'consonant', all vector entries are 1 (abbr. GCT-all1). With regards to tonal consonance GCTs, both the degrees and the chord types are described. The first part explains the position of the chord in the scale, while the latter the intervals comprising the chord.

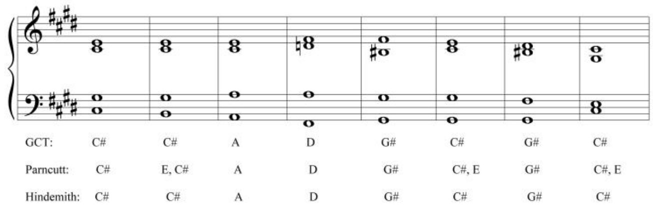

Figure 1 Reduction of m.1-5 of Beethoven's Piano Sonata op.27 no.2.

Obviously, it is ineffective to analyse tonal music harmonically with set theory. It is claimed that octave equivalence and inversional equivalence are shared features of all – at least Western music – idioms' analysis. However, inversion is used slightly differently in the two contexts. For instance, in atonal music it refers to setting the order of intervals of a pitch-class set in reverse (Kostka, 2006). Whereas in tonal idioms, or idioms that have a tonal component, it seems that the order of intervals in a pitch simultaneity is important.

Table 1 List of representations for Beethoven op.27 no.2.

PCs	Forte sets	GCT	GCT_all1
1 4 8	0 3 7	0 0 3 7	0 0 3 7
1 4 8 11	0 3 5 8	0 0 3 7 10	7 0 3 5 8
1 4 9	0 3 7	8 0 4 7	8 0 4 7
2 6 9	0 3 7	1 0 4 7	1 0 4 7
0 6 8	0 2 6	7 0 4 10	5 0 2 6
1 4 8	0 3 7	0 0 3 7	0 0 3 7
0 3 6 8	0 2 5 8	7 0 4 7 10	11 0 3 6 8*
1 4 8	0 3 7	0 0 3 7	0 0 3 7

Apart from atonal music, where tonality is non-existent, one can discover tonal centres in other non-tonal idioms. Kostka (2006) calls the method to establish a tonal centre in such works, tonic by assertion, and is achieved by "the use of reiteration, return, pedal point, ostinato, accent, formal placement, register, and similar techniques" (p.102).

The next two examples, figure a tonal centre, so it can be used as a reference in a pitch hierarchy. In the Hindemith excerpt (Fig. 2), a B appears as a drone tone in the soprano, while there is a melodic movement around E in the bass. E is picked between the two as the main pitch reference, and the pitch hierarchy suggested is E mixolydian, or in GCT notation 4 [0,2,4,5,7,9,10].

The excerpt from Scriabin's etude (Fig. 3) is a bit more complex. There are two whole tone scales in m. 1,3-4 and m. 2 respectively. It is very hard to say which are the bases of those two scales. The chromatic scale can be a common reference point for the whole excerpt, also because it deals with the interchange between two scales in such a short period.

Figure 2 Reduction of m.1-3 of Hindemith's choral song *Un cygne* from the *Six Chansons*.

The main difference between GCT's roots and roots proposed by Hindemith is on chords built on fourths. In case of quartal chords, GCT picks the note placed in the lowest consecutive fourths, a somewhat appropriate decision, since there is a principle to always output a solution (Cambouropoulos, Kaliakatsos-Papakostas, Tsougras, 2014).

In Table 2 below, as well as in Table 1 and the rest of the tables, we can observe many similarities between the Forte sets and GCT_all1. The differences are noted with an asterisk (*) next to the GCT representation. The most usual issue is with regards to combinations that include major or minor triads. For instance, a major triad [0,4,7] is a subset of [0,2,4,7], while its GCT version is [7,[0,3,5,7]] (see chord no. 9).

Table 2 List of representations of Hindemtih's *Un cygne*.

PCs	Forte sets	GCT	GCT_all1
4 9 11	0 2 7	7,0 5 10	5,0 2 7
2 7 9 11	0 2 4 7	3,0 4 7 14	3,0 2 4 7
1 6 8 11	0 2 5 7	4,0 5 10 15	2,0 2 5 7
4 6 11	0 2 7	2,0 5 10	0,0 2 7
2 4 9 11	0 2 5 7	7,0 5 10 15	5,0 2 5 7
4 6 11	0 2 7	2,0 5 10	0,0 2 7
1 6 8 11	0 2 5 7	4,0 5 10 15	2,0 2 5 7
1 2 6 9	0 1 5 8	10,0 4 7 11	9,0 1 5 8
2 4 6 11	0 2 4 7	7,0 3 7 17	7,0 3 5 7*
2 6 9	0 3 7	10,0 4 7	10,0 4 7
1 4 6 11	0 2 5 7	9,0 5 10 15	7,0 2 5 7
7 11	0 4	3,0 4	3,0 4
6 7 11	0 1 5	7,0 7 13	2,0 1 5

Hindemith's harmonic language extensively uses quartal and quintal chords. When the [0,5,10] representation is replaced by [0,2,7], the concept of superimposed fourths becomes less obvious. Taking into account his theoretical background, the first chord would not have a root and be part of group V, but the other version fits in group III, thus revoicing would impact a chord's effect. By abstracting it that much, the factor of the positioning of intervals is ignored, an issue that is important in the composer's music. Also perceptionally those two differ significantly. A sus2 chord has a completely different sound than a quartal. There is no debate about the consonance of the perfect 5th, as opposed to two stacked perfect 4^{ths}.

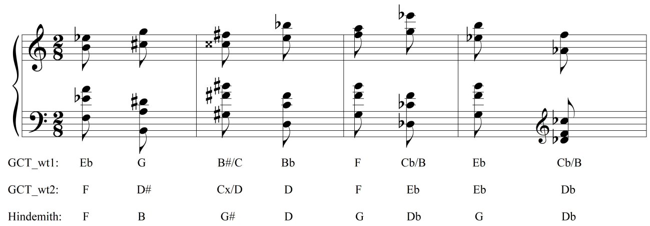

Figure 3 Reduction of m.1-4 of Scriabin's Etude op. 56 no.4

For the analysis of excerpt from Scriabin's Etude, Parncutt's perceptual pitch model gives in almost all cases two equal maximum values, which are greater than the rest by 2 or 3 units. This ambiguity makes it more difficult to compare the results with the other models.

With regards to GCT, it is tested with two possible variations of the consonance / dissonance vector. We suppose, based upon the interval vector of the whole tone scale <060603>, that the existing intervals should be 'consonant'. So the resulting vectors are [1,0,1,0,1,0,1,0,1,0,1,0] and [1,0,0,0,1,0,1,0,1,0,0,0] where the first considers 'consonant' unison, major 2nd, major 3rd, tritone, minor 6th and minor 7th, and the second the same without major 2nd and minor 7th.

The first vector (named here as wt1, and the other wt2) struggles with two issues. When compared with GCT_all1, they share the same representation, bases included, except from chords no. 6 and 8 (see Table 3). Apart from that, it doesn't share any root suggested by Hindemith's theoretical approach either.

On the other hand, wt2 regards major 3rd of higher importance as an element in encoding. We can see some types of whole-tone chords in Kostka (2006). The proposed encodings by GCT are very similar to them. Major 3rd is indeed the basis of their construction, however when they have 4 or more voices, major 2^{nds} are inevitable. Kostka avoids to write major 2^{nds} near bass, and at the same time keeps the chord in the closest form possible so it's built on 3^{rds}.

Table 3 List of representations of Scriabin Etude op.56 no. 4.

PCs	Forte	GCT_all1	GCT_wt1	GCT_wt2
3 5 9 11	0 2 6 8	3,0 2 6 8	3,0 2 6 8	5,0 6 10 16
1 3 7 9 11	0 2 4 6 8	7,0 2 4 6 8	7,0 2 4 6 8	3,0 4 8 10 18
0 2 6 8	0 2 6 8	0,0 2 6 8	0,0 2 6 8	2,0 6 10 16
0 2 4 6 10	0 2 4 6 8	10,0 2 4 6 8	10,0 2 4 6 8	2,0 4 8 10 14
5 7 9 11	0 2 4 6	5,0 2 4 6	5,0 2 4 6	5,0 4 6 14
1 3 5 7 11	0 2 4 6 8	10,0 2 4 6 8	11,0 2 4 6 8	3,0 4 8 10 14
3 5 7 11	0 2 4 8	3,0 2 4 8	3,0 2 4 8	3,0 4 8 14
1 5 8 11	0 2 5 8	5,0 3 6 8	11,0 2 6 9	1,0 4 10 19

For the last example, a rather extreme case has been examined. Clearly it is meaningless to look for 'roots' in an atonal piece, designed not to have such. Hindemith's theory, however, can propose encodings even for the most complex chords. Surprisingly, tonal GCT, Hindemith and Parncutt agree with each other, with few exceptions. GCT_all1, however, gives very different results. GCT_all1 seems to be closely related to the prime forms in Forte's theory (GCT_all1 produces a normal order encoding accompanied with a transposition operator).

Figure 4 m.1-4 of Webern's choral song Entflieht auf leichten Kähnen op.2.

Table 4 List of representations of Webern's Entflieht auf leichten Kähnen op.2.

PCs	Forte sets	GCT	GCT-all_1
2 5 9 11	0 2 5 8	2,0 3 7 9	9,0 2 5 8
0 4 6 10	0 2 6 8	0,0 4 6 10	4,0 2 6 8
2 7 11	0 3 7	7,0 4 7	7,0 4 7*
0 4 8	0 4 8	0,0 4 8	0,0 4 8
1 5 9 10	0 1 4 8	10,0 3 7 11	9,0 1 4 8
4 5 9	0 1 5	5,0 4 11	4,0 1 5
4 6 7 10	0 2 3 6	4,0 3 6 14	4,0 2 3 6
2 6 7 11	0 1 5 8	7,0 4 7 11	6,0 1 5 8
1 2 8 10	0 1 4 6	10,0 3 4 10	8,0 2 5 6*
0 4 9	0 3 7	9,0 3 7	9,0 3 7
1 8 10	0 2 5	10,0 3 10	8,0 2 5
4 7 10	0 3 6	4,0 3 6	4,0 3 6
2 6 8 11	0 2 5 8	11,0 3 7 9	6,0 2 5 8
1 5 8 9	0 1 4 8	1,0 4 7 8	5,0 3 4 8*
0 5 9	0 3 7	5,0 4 7	5,0 4 7*
1 5 7 10	0 2 5 8	10,0 3 7 9	5,0 2 5 8
3 7 10 11	0 1 4 8	3,0 4 7 8	7,0 3 4 8*
2 8 10 11	0 2 3 6	8,0 3 6 14	8.0 2 3 6
2 5 9 10	0 1 5 8	10,0 4 7 11	9.0 1 5 8

V. CONCLUSION

Tonal ambiguity in non-tonal – even in some tonal – contexts has been a hard issue to resolve, since the conception of 'chord' and 'root'. For sure, naming roots considering the general existing hierarchies might lead to fallacies, like in atonal music. Nevertheless, when we decide to encode a symmetrical or a complex non-tertian chord it is necessary to reach a 'reasonable' solution. Topics in categorical perception make the whole enterprise of encoding chords an interesting problem.

GCT representation works effectively in tonal idioms, when tested against standard harmonic ground truth data or compared with other models, such as Parncutt's perceptual root model. As far as Hindemith's neo-tonal music, the representation of quartals was sufficient also with regards to maintaining the order of intervals in a chord. For different pitch hierarchies, like the whole-tone idiom, it is not trivial to encode chords effectively. Interval vectors can be a useful tool to deal with them. Seconds, when considered 'consonant' in the above examples, had the tendency to preoccupy the representations in comparison to other intervals. This can be seen between GCT_all1 and GCT_wt1.

Apart from those idioms, GCT works well in atonal music. The flexibility of the consonance/dissonance vector, makes GCT_all1 similar to Forte's prime forms (literally identical to Tn-transposition-related normal orders). With regards to Hindemith's theory, it isn't accepted by many theorists and can only be loosely applied in atonal settings. Parncutt's perceptual root-finding model was primarily designed for tonal music, but it might be extended if empirical results come up from research similar to that of Krumhansl and Kessler (1982).

Forte's prime forms are based on the structure of intervals in a pitch class set, omitting the need to a referential point. However, it seems necessary, when the existing tonality, whichever its use, is taken into account. Although, it is uncertain whether the strict mathematical abstraction of Forte's prime forms or the more generic one of GCT fits better at the concepts of categorical perception suggested in the beginning.

ACKNOWLEDGMENT

I would like to thank Maximos Kaliakatsos-Papakostas for his help about the algorithmic applications and Costas Tsougras for his theoretical insights.

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