ΤΥΠΟΛΟΓΙΟ
ΕΦΑΡΜΟΣΜΕΝΗΣ ΣΤΑΤΙΣΤΙΚΗΣ
Χρόνης Μουσιάδης

ΠΟΛΛΑΠΛΗ ΠΑΛΙΝΔΡΟΜΗΣΗ

\[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon \]

\[ \hat{Y} = X \hat{\beta} + \varepsilon \]  

\[ \hat{\beta} = (X'X)^{-1}X'Y \]

\[ SS = Y'(X'X)^{-1}X'Y - \frac{1}{n} \sum Y \]

\[ SST = Y'(I_n - \frac{1}{n} \sum 1)Y \]

\[ SSR = Y'(X'X)^{-1}X'Y - \frac{1}{n} \sum Y \]

\[ SST = SS + SSE \]

\[ I \frac{SSR}{\sigma^2} \sim \chi^2_{k} \quad \text{with} \quad \lambda = \frac{1}{n} \beta' \left[ X'(I_n - \frac{1}{n} \sum 1)X \right] \beta \]

\[ \frac{1}{\sigma^2} SSE \sim \chi^2_{n-k-1} \quad \text{with} \quad \lambda = 0 \]

\[ \frac{1}{\sigma^2} SST \sim \chi^2_{n-1} \quad \text{with} \quad \lambda = \frac{1}{n} \beta' \left[ X'(I_n - \frac{1}{n} \sum 1)X \right] \beta \]

\[ MSR = \frac{SSR}{k} \]

\[ MSE = \frac{SSE}{n-k-1} \]

\[ F = \frac{MSR}{MSE} \sim F_{k,n-k-1} \]

\[ Var(\hat{\beta}_i) = s^2 \gamma_{i,i} \quad \text{for} \quad i = 0, 1, \ldots, k \]

\[ s(\hat{\beta}) = s \sqrt{\gamma_{0,0}} \quad \text{for} \quad i = 0, 1, \ldots, k \]

\[ Var(\hat{\gamma}_0) = s^2 \gamma_{0,0} \]

\[ C = (X'X)^{-1} = \begin{pmatrix}
    c_{00} & c_{01} & \ldots & c_{0k} \\
    c_{10} & c_{11} & \ldots & c_{1k} \\
    \vdots & \vdots & \ddots & \vdots \\
    c_{k0} & c_{k1} & \ldots & c_{kk}
\end{pmatrix} \]

\[ R = D_s^{-1/2} D_s^{-1/2} \]

\[ D_s = \text{diag}(S) \]

\[ R_{11} \sim \chi^2_{k} \quad \text{with} \quad \lambda = \frac{1}{n} \beta' \left[ X'(I_n - \frac{1}{n} \sum 1)X \right] \beta \]

\[ R_{11} > R_0 \quad \text{for} \quad R_0 = 1 - (1 - R^2) \left( 1 + \frac{k}{n-k-1} F_{k,n-k-1} \right) \]

\[ c_p = \frac{SSE_p}{s^2} - (n-2p) \]

\[ \hat{Y}_k \quad \text{for} \quad k = 1, 2, \ldots, p \]

\[ \hat{Y}_k = \sum_{j=1}^{q} Y_{jk} \quad \text{for} \quad j = 1, 2, \ldots, g \]

\[ \hat{Y}_i = \sum_{k=1}^{p} Y_{ik} \quad \text{for} \quad i = 1, 2, \ldots, f \]

\[ \hat{Y} = \sum_{i=1}^{f} \sum_{j=1}^{q} \sum_{k=1}^{p} Y_{ijk} = \sum_{i=1}^{f} Y_{i} \quad \text{for} \quad \gamma = 1 \]

\[ \hat{Y}_i = \frac{1}{q^r} Y_{i} \quad \text{for} \quad \gamma = 1 \]

\[ \hat{Y}_k = \frac{1}{p^g} Y_{k} \quad \text{for} \quad \gamma = 1 \]
Ένας Παράγοντας με \( p \) στάθμες και \( r_i \) μετρήσεις στη στάθμη \( i \):

\[
SSA = \sum_{i=1}^{n} \frac{Y_i^2}{r_i} - \bar{Y}^2, \quad SST = \sum_{j=1}^{n} \sum_{i=1}^{r_j} \frac{Y_{ij}^2}{n} - \bar{Y}^2, \quad n = \sum_{i=1}^{p} r_i
\]

Δύο Παράγοντες με \( p, q \) στάθμες (\( p \times q \)) και με \( n \) μετρήσεις σε κάθε κελι

\[
SSA = \sum_{i=1}^{p} \frac{Y_i^2}{qn} - \frac{Y^2}{pqn}, \quad SSAB = \sum_{i=1}^{p} \sum_{j=1}^{q} \frac{Y_{ij}^2}{n} = \sum_{j=1}^{q} \frac{Y_j^2}{qn} - \sum_{j=1}^{q} \frac{Y_j^2}{pn} + \frac{Y^2}{pqn}
\]

Τρεις Παράγοντες με \( p, q, r \) στάθμες (\( p \times q \times r \)) και με \( n \) μετρήσεις σε κάθε κελι

\[
SSA = \sum_{i=1}^{p} \frac{Y_i^2}{qrn} - \frac{Y^2}{pqrn}, \quad SSA = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} \frac{Y_{ijk}^2}{n} - \sum_{j=1}^{q} \sum_{k=1}^{r} \frac{Y_{jk}^2}{pn} + \frac{Y^2}{pqrn}
\]

ΧΡΟΝΟΣΕΙΡΕΣ

\[
\gamma_k = \text{Cov}(z_{i+k}, z_{i+k}) = E(z_i z_{i+k}) \quad \rho_k = \frac{\gamma_k}{\gamma_0}, \quad r_k = \frac{C_k}{C_0} \quad \text{και} \quad r_k - \frac{N(\rho_k, 1/N)}
\]

\[
\Gamma_N = \begin{bmatrix} \gamma_0 & \gamma_1 & \gamma_2 & \ldots & \gamma_N-1 \\ \gamma_1 & \gamma_0 & \gamma_1 & \ldots & \gamma_N-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{N-2} & \gamma_{N-3} & \gamma_{N-4} & \ldots & \gamma_0 \end{bmatrix}, \quad P_N = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \ldots & \rho_{N-1} \\ \rho_1 & 1 & \rho_1 & \ldots & \rho_{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{N-2} & \rho_{N-3} & \rho_{N-4} & \ldots & 1 \end{bmatrix}
\]

\[
c_k = \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z})(z_{i+k} - \bar{z}), \quad k = 0, \pm 1, \pm 2, \ldots
\]

### GLM

\[
z_i = \alpha_i + \psi_1 \alpha_{i-1} + \psi_2 \alpha_{i-2} + \ldots
\]

### AR(p)

\[
z_i = \Phi_1 z_{i-1} + \Phi_2 z_{i-2} + \ldots + \Phi_p z_{i-p} + \alpha_i \quad \Phi(B) z_i = \alpha_i
\]

\[
\rho_k = \Phi_1 \rho_{k-1} + \Phi_2 \rho_{k-2} + \ldots + \Phi_p \rho_{k-p}, \quad k \geq 0
\]

### MA(q)

\[
z_i = \alpha_i - \theta_1 \alpha_{i-1} - \theta_2 \alpha_{i-2} - \ldots - \theta_q \alpha_{i-q} \quad z_i = \Theta(B) \alpha_i
\]

\[
\rho_k = \left\{ \begin{array}{ll} 0, & k > q \\ -\theta_1 \rho_{k+1} - \theta_2 \rho_{k+2} - \ldots - \theta_q \rho_{k-q}, & k = 1, 2, \ldots, q \\ 1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2, & k < 0 \\ \rho_{k+1}, & k \geq 0 \end{array} \right.
\]

### ARMA(p,q)

\[
z_i = \Phi_1 z_{i-1} + \ldots + \Phi_p z_{i-p} + \alpha_i - \theta_1 \alpha_{i-1} - \ldots - \theta_q \alpha_{i-q} \quad \Phi(B)z_i = \Theta(B)\alpha_i
\]

### ARIMA(p,d,q)(P,D,Q)_k

\[
\phi_p(B) \Phi_p(B^d) \phi_q(B^q) z_i = \theta_q(B) \Theta_q(B^q) \alpha_i
\]

\[
Q = N \sum_{i=1}^{N} r_i^2 (\hat{\alpha}_i) \sim \chi^2_{p-q} \quad AIC(K) = N \log \hat{\sigma}^2 + K
\]

### Ανάλυση Διασποράς

<table>
<thead>
<tr>
<th>ΠΗΓΗ</th>
<th>ΑΘΡΟΙΣΜΑΤΑ ΤΕΤΡΑΓΩΝΩΝ</th>
<th>β.ε.</th>
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<tbody>
<tr>
<td>ΓΡΑΜΜΕΣ</td>
<td>( \sum_{i=1}^{n} Y_i^{2} - \bar{Y}^2 )</td>
<td>( t-1 )</td>
</tr>
<tr>
<td>ΣΤΗΛΕΣ</td>
<td>( \sum_{i=1}^{n} Y_i^{2} - \bar{Y}^2 )</td>
<td>( t-1 )</td>
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<tr>
<td>ΓΡΑΜΜΑΤΑ</td>
<td>( \sum_{i=1}^{n} Y_{ij}^{2} - \bar{Y}_{ij}^2 )</td>
<td>( t-1 )</td>
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<tr>
<td>ΥΠΟΛΟΙΠΑ</td>
<td>Με Αφαίρεση</td>
<td>( (t-1)(t-2) )</td>
</tr>
<tr>
<td>ΣΥΝΟΛΟ</td>
<td>( \sum_{i=1}^{n} \sum_{j=1}^{r} Y_{ijk}^2 - \bar{Y}_{ijk}^2 )</td>
<td>( t^2 - 1 = n-1 )</td>
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<tr>
<td>Αθροιστική συνάρτηση κατανομής της κανονικής N(0,1) κατανομής</td>
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<thead>
<tr>
<th>Κρίσιμες τιμές της κατανομής (t_n) για στάθμη σημαντικότητας (\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
</tr>
<tr>
<td>(t_n)</td>
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<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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</table>

Σε όλες τις πινακίδες (πίνακας όπως και οι επόμενοι τρεις κατασκευάστηκαν με το πρόγραμμα S-Plus, με τις εντολές (αντίστοιχα):

- matrix(pnorm(seq(0.3,49.01)),ncol=10,byrow=T)
- a<-c(0.75,9.95,7.95,9.95,9.995)
- b<-c(1:30,40,60,120,1000)
- matrix(qt(rep(a,4),rep(b,each=7)),ncol=7,byrow=T)
- a<-c(0.005,0.025,0.1,0.25,0.5,0.75,0.95,0.975,0.995)
- matrix(qchisq(rep(a,3),rep(1:30,each=13)),ncol=13,byrow=T)
- n<-c(1:10,12,15,20,30,60,120,1000)
- qf(c(0.95,0.99),rep(n,2),each=2,rep(n,each=34))
| n | 1.045 | 1.000 | 0.997 | 0.995 | 1.000 | 0.997 | 0.995 | 1.000 | 0.997 | 0.995 | 1.000 | 0.997 | 0.995 | 1.000 | 0.997 | 0.995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 2 | 0.0101 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 |
| 3 | 0.0077 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 |
| 4 | 0.0015 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 |
| 5 | 0.0080 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 |

95% και 99% άνοι προσοχή σημεία της κατανομής $\chi^2$ με στάθμη σημαντικότητας $\alpha$