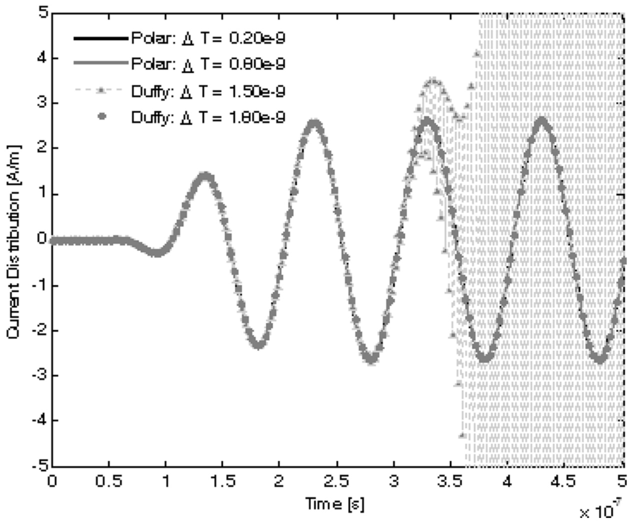

Fig. 8. This figure shows an example run on a unit cube. Polar quadrature enables stability at a much smaller time step than Duffy's method.

the integrand is rendered non-singular, and the integration becomes one dimensional. In addition, polar integration is equally applicable to all interactions, near and far. Finally, because the numerical integration is one dimensional, it is easy to adaptively integrate the potentials to arbitrary accuracy.

As the authors were preparing for final submission upon completion of the review process for this paper, a similar paper [13] entered the literature. While the technique is very similar to the method presented in this paper, it does not address the temporal shape of potential curves as related to stability. Therefore, we believe this paper still represents a significant contribution to the community.

ACKNOWLEDGMENT

The authors would like to thank J. Rockway, B. Fasenfest, and D. White of Lawrence Livermore National Laboratory for their assistance in testing the ideas presented in this paper.

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Managing Topological Prioritization in Ray-Tracing Based Progressive Propagation-Prediction Modeling

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***Abstract*—In a progressive propagation prediction model, significant time is saved by interrupting the process when specific application related constraints have been met. Such limitations have to do with the statistics of intermediate results as well as their spatial arrangement in the area of interest. We propose a time-efficient method to control the topological distribution of intermediate predictions. The proposed criterion allows different priorities to be set in different regions of the study area. In addition, we put forward two progressive strategies capable of adjusting the ray-tracing engine to the desired prioritization. The advantages of the proposed strategies compared with existing approaches are well demonstrated.**

***Index Terms*—Progressive propagation modeling, radiocoverage, ray-tracing.**

I. INTRODUCTION

Traditional empirical or semi-deterministic propagation prediction models have been replaced by deterministic ones based on a combination of geometrical optics and the uniform theory of diffraction, [1]–[3]. The running-time of such models is very important, as mirrored in the efforts after the early 1990s that focus on accelerating the performance of the early models. At the same time, different applications require diverse characteristics concerning the predictions delivered by these models. Significant time could be saved if such a prediction process could be stopped when specific demands, regarding the final results have been safely satisfied.

In a progressive prediction model, intermediate results are continuously fed back to the user, who can stop the prediction process when the specific-application-related required constraints have been met. The concept of progressiveness in ray-tracing propagation prediction models was introduced in [4]. The authors set the basic rules that govern such models. They state that ray-paths should be traced in

Manuscript received September 21, 2005; revised April 5, 2006.

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Digital Object Identifier 10.1109/TAP.2006.882197

such a manner that both the mean and the variance of prediction errors should decrease monotonically. Through the faithfulness and fairness theorems, they demonstrate that these requirements are met, if for each receiver's location, ray-paths are traced according to their non-increasing contribution ratios to the received signal strength. In addition, a workload estimator has been put forward, to ensure that the process stops when the statistics of the error satisfy a given threshold.

Even though they claim that intermediate results should be fair and unbiased for all receivers, they do not provide a means to examine and evaluate their spatial distribution. We put forward a new criterion to control the topological distribution of intermediate predictions. The proposed test is fast and suitable for any arrangement of receivers in the propagation area. Furthermore, we allow the user to set diverse priorities in different regions of the area of interest. This flexibility is particularly important for certain applications, such as planning, where the region near the coverage limits could be of greater interest than that near the transmitter, due to the expected interaction with the neighboring cells and the related smaller power-values.

In addition, we propose two progressive strategies that adjust to the diverse topological prioritizations of each application, and compare them with the Source-Group-Raypath-Interleave (SGRI) algorithm (that demonstrated the best behavior in [4]), in two scenarios; one assuming even interest in the entire study area and one with a prioritized region. In both cases one of the proposed strategies outperforms the SGRI in all statistics (mean error, standard deviation, topological distribution of predictions). The proposed criterion is presented in Section II. Then, the progressive strategies are briefly described in Section III. All strategies are compared in Section IV and we conclude in Section V.

II. Topological Prioritization in Progressive Predictions

At first, let us assume that a progressive model is desired to deliver intermediate results uniformly arranged in the study area. At a given time t, intermediate estimations of the received field are performed in M out of N study points, $M \leq N$. Literally, we seek an appropriate selection of M out of N sites, in such a manner that an estimation of the final result has been obtained close to, or exactly at each one of the N study-points. In mathematical terminology, we search for the M out of N study-points that minimize the sum of the distances from all study points to the nearest position where estimation has been delivered. Let us consider N study points, the positions of which are represented with the vectors $\boldsymbol{z_i}$, $i = 1, \ldots, N$. We denote as

$$L_j = [\boldsymbol{x}_{1j} \quad \boldsymbol{x}_{2j} \quad \ldots \quad \boldsymbol{x}_{Mj}]^T \tag{1}$$

the j_{th} out of $\binom{N}{M}$ different selections of M out of N study points. Then $\boldsymbol{x}_{ij}$, $i = 1, \ldots, M$ represent the positions of the M calculation points of this selection in vector notation, and could be any of the N study points $\boldsymbol{z}_i$. For each L_j, we calculate

$$D_j = \sum_{i=1}^{N} \left|\boldsymbol{z}_i - \boldsymbol{x}_{\lambda_{\mathrm{opt}}j}\right| \tag{2}$$

the sum of the distances, denoted as "$|\bullet|$," of the positions of the N study points $\boldsymbol{z}_i$ from the closest calculation point of the set L_j, that is for each $\boldsymbol{z}_i$, we search for the $\boldsymbol{x}_{\lambda j}$, $\lambda = 1, \ldots, M$ that minimizes the distance "$|\boldsymbol{z}_i - \boldsymbol{x}_{\lambda j}|$":

$$\lambda_{\mathrm{opt}} = \arg \min_{\lambda=\{1,2,\ldots,M\}} \{|\boldsymbol{z}_i - \boldsymbol{x}_{\lambda j}|\} \tag{3}$$

where $\arg \min_{\lambda=\{1,2,\ldots,M\}} \{\bullet\}$ means "select the argument λ out of the $\{1, 2, \ldots, M\}$ that minimizes $\bullet$." Then, out of the $\binom{N}{M}$ different L_k we select L as

$$L = L_{k_{\mathrm{opt}}}, \text{where } k_{\mathrm{opt}} = \arg \min_{k=\left\{1,2,\ldots,\binom{N}{M}\right\}} \{D_k\}. \tag{4}$$

Moving one step further, one specific area could be of greater interest than another. One could express the different priorities in the calculations by properly weighting each study-point, assigning greater positive values in points of greater interest. In such a case, the selection of the M study points can be still delivered by (1)–(4), by substituting (2) with:

$$D_j = \sum_{i=1}^{N} w_i \left|\boldsymbol{z}_i - \boldsymbol{x}_{\lambda_{\mathrm{opt}}j}\right| \tag{5}$$

where w_i is the weight assigned to $\boldsymbol{z}_i$. Since we ask for the minimization of (5), the greater the weight is, the greater the importance of the specific term in the above sum and hence the solution will include a calculation point near $\boldsymbol{z}_i$, to minimize the contribution of the specific term in (5).

The application of (1)–(5) in a progressive propagation prediction planning tool is impractical, due to the required running time to perform the exhaustive search of all possible $\binom{N}{M}$ solutions. However, a good estimation of the final solution can be obtained by implementing the k-means algorithm [5]. The algorithm partitions the area into M clusters, so that the total distance between all study-points and their centroids is minimized. The M centroids represent the optimum sampling points. The algorithm converges fast to a "local" minimization of (2) [or (5)] that depends on the initial conditions. One can improve the estimation, by repeating the applications of k-means several times with different initial conditions and selecting the solution that minimized (2) [or (5)].

In the following section, we present two progressive algorithms and compare them with the SGRI algorithm proposed in [4]. We evaluate both the topological distribution of intermediate predictions as well as the statistics of the error; defined as the difference in dB between the intermediate predictions and the final result.

III. Progressive Prediction Strategies

A brute-force ray-tracing method, presented in [6], is used in our model. All secondary sources of radiation are identified in a pre-processing stage. Rays are launched from each source, as described in [6], at small angular separation. After an intersection, a transmitted and a reflected ray-segment are generated and traced. The process is repeated until the ray-segment exits the study area or the magnitude of the field falls below a predefined threshold (no reflection number limit is considered). This threshold, defined by the user, represents the noise-level of the receiver.

The differences of the following three algorithms lie in the prioritization of either the ray-launching procedure or the processing of the ray-segments.

A. SGRI Algorithm [4]

- The rays from the real sources [antenna(s)] are launched and processed first.
- If many sources (either real or secondary) exist, they are partitioned into different geometric regions, based on their geometric locations.
- Ray-paths within the same region are interleaved.

- Ray-paths from different regions are clustered into groups.

B. Proposed Algorithms

The main difference in the proposed algorithms, compared to the ones suggested in [4] is that we do not "*a priori*" assign a "high-priority" value to the real sources. In fact, a real source could be given the lowest priority, depending on the application. Of course a real source is always indirectly included in the calculations that regard a secondary source (e.g., in a "diffraction," the characteristics of the transmitted field are included in the calculations of the diffracted one). Priorities are assigned in an "illumination-based" manner.

We assume that each study point, whose position is defined by $\boldsymbol{z}_i$, is assigned a non-negative priority-value $w_i(\boldsymbol{z}_i)$. Furthermore, each source S_i, either real or secondary, is also given a priority value $p_i(S_i)$. If different priorities are considered at diverse regions of the study area, each source is assigned the largest priority value among the receivers that it unobstructively illuminates. Assuming that $C_j(S_j)$ is the subset of receivers that the source S_j unobstructively illuminates, then $p_j(S_j) = \max\{w_k(\boldsymbol{z}_k) | \boldsymbol{z}_k \in C_j(S_j)\}$.

As a consequence, the priority-values assigned to the sources maintain the ratio of the values assigned to the corresponding receivers. If no prioritization is considered in the study area, all sources are grouped together $(p_i(S_i) = \mathrm{constant}, \forall i)$.

1) Algorithm I:

- Priorities are assigned to each source after performing a simple illumination test from each receiver to all sources.
- Sources with the same priority value p_i are grouped together.
- The number of successive rays considered from each priority group is proportional to the product of the number of sources of the group with their corresponding priority values. For example, if two groups are assigned priority-values 1 and 3, and contain 10 and 5 sources respectively, then for 10 generated rays from the low priority group 15 rays will be launched from the high priority group.
- For each group, successive rays are launched from different sources.
- Each ray and all its children (reflected and transmitted rays) are traced until they exit the study area or the magnitude of the field falls below the predefined threshold.

2) Algorithm II:

- Priority assignment and ray-launching is performed as in Algorithm I.
- The contribution of each generated ray to the study points is calculated until an intersection has been reached. The transmitted and reflected rays are stored in a buffer.
- When all the source-generated rays from the same priority-group are finished, the previously stored ray-segments are treated until the next intersection. The stored ray-segments maintain their "priority-values," thus preserving the same rate of calculations for the different priority groups.
- The process stops, when all ray-segments have been calculated.

IV. Results-Discussion

For the evaluation of the proposed algorithms, we have considered the transmission of a 1.8 GHz carrier at 30 dBm from a theoretical isotropic antenna located well below rooftop level in a typical urban environment with straight streets of arbitrary widths, as depicted in Fig. 1. The receiver's threshold is considered at -110 dBm. A 2-D ray-launching process is performed. Only 1st-order diffraction is considered. The total number of 1st order diffraction sources is 28. The study points are arranged at 5 m intervals outside of the buildings, resulting in a total of 2290 mobile-positions. Part of this arrangement is shown at the bottom left side of Fig. 1. Rays are launched at an angular separation of 0.03°. Simulations were carried out by a desktop PC, with a Pentium IV processor at 2.8 GHz, 1048 MB RAM, operating under Microsoft Windows XP.

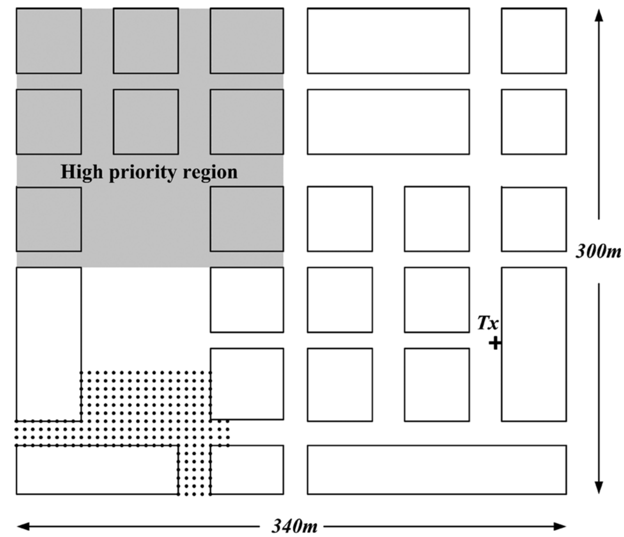

Fig. 1. Typical urban configuration, representing the simulation-scene.

The process was monitored at small time-intervals. Intermediate predictions are delivered at a study point as soon as a ray crosses it. Some characteristic results are summarized in Table I. Even interest was assumed in the entire study area. In order to evaluate the performance of the three algorithms in terms of the topological distribution of intermediate predictions, the following parameter was calculated, in conformance with (2):

$$D' = \sum_{i=1}^{N} |\boldsymbol{z}_i - \boldsymbol{y}_{\lambda_{\mathrm{opt}}}|, \text{ with } \lambda_{\mathrm{opt}} = \arg \min_{\lambda=\{1,2,\ldots,M\}} \{|\boldsymbol{z}_i - \boldsymbol{y}_\lambda|\} \quad (6)$$

where $\boldsymbol{y}_\lambda$ are the M study-points, where an estimation has been delivered. Apart from (6), the k-means routine is invoked with the number of clusters equal to M that provides a good estimation of (4), (5); that is an estimation of the optimum arrangement of the M points, where prediction has been delivered at a given time t, in order to minimize (5) (or (2) when all weights equal 1). The achieved estimation of (5), obtained with k-means is represented as D_{means}. Then, D' is compared with D_{means} through

$$D_{\mathrm{diff}} = D' - D_{\mathrm{means}} \quad (7)$$

$$D_{\mathrm{diff}}(\%) = \frac{D' - D_{\mathrm{means}}}{D_{\mathrm{means}}} 100\% \quad (8)$$

(7) and (8), as well as the percentage of the area where an estimation has been delivered (Cov. %) are given in Table I and are considered sufficient criteria to evaluate the performance of the 3 algorithms in terms of the topological distribution of intermediate field strength predictions in the study area. In the same table, the statistics of the difference of the intermediate prediction from the final (mean $\hat{e}$ and standard deviation σ) are also given. In a real application of a progressive strategy, the above statistics can only be estimated, by implementing the workload estimator, presented in [4].

The 1st result, obtained at $10''$, shows a very poor performance of the SGRI in all categories. At that interval, only the real source has been considered in the SGRI case. As a consequence, the topological distribution of intermediate predictions is concentrated around the transmitting antenna, as mirrored in (7) and (8).

TABLE I
COMPARISON OF PROGRESSIVE STRATEGIES (NO PRIORITY)

Time (s)	Cov. (%)	$D_{\rm diff}$ (m)	$D_{\rm diff}$ (%)	Mean (dB)	Dev. (dB)
SGRI					
10.03	53.95	24886.1	461.19	34.95	17.21
21.25	92.44	149.64	16.82	9.11	11.08
41.69	95.41	62.92	11.93	5.10	9.05
62.18	96.76	52.07	14.07	3.60	7.47
82.28	97.16	40	12.12	2.91	6.74
107.69	97.95	5	2.13	2.27	6.13
179.28	98.86	-	-	1.11	4.45
235.32	99.12	-	-	0.76	4.05
415.05	100.00	-	-	0.00	0.00
ALGORITHM I					
10.29	92.14	137.93	15.22	9.07	11.03
20.45	94.19	85.86	12.74	6.56	9.94
40.78	96.12	45.86	10.05	4.34	8.59
61.16	97.03	22.6	6.2	3.18	7.12
81.62	97.6	20	7.27	2.55	6.38
106.92	98.16	-	-	1.99	5.82
178.14	98.95	-	-	0.99	4.32
234.11	99.12	-	-	0.71	3.92
411.75	100.00	-	-	0.00	0.00
ALGORITHM II					
10.03	83.33	1281.12	66.47	5.02	9.1
20.06	87.69	1045.49	73.5	4.64	8.81
40.15	90.83	1007.27	95.55	4.47	8.72
61.42	91.88	984.87	105.66	4.3	8.59
81.48	92.1	975.2	107.26	4.15	8.48
106.56	93.8	727.68	102.49	2.96	8.11
178.76	96.2	471.6	107.39	1.15	6.41
235.86	97.77	267.42	103.2	0.63	5.34

TABLE II
COMPARISON OF PROGRESSIVE STRATEGIES (PRIORITIZED)

Time (s)	Cov. (%)	$D_{\rm diff}$ (m)	$D_{\rm diff}$ (%)	Mean (dB)	Dev. (dB)
SGRI					
10.03	54.17	28867.2	571.06	36.18	16.71
20.89	92.58	231.4	33.25	10.28	11.23
41.45	95.5	109.24	26.13	5.82	8.86
61.83	96.77	76.26	25.24	4.3	7.61
87.15	97.42	49.42	20.49	3.45	6.96
97.2	97.77	31.9	15.4	2.96	6.43
153.02	98.52	-	-	1.6	4.69
213.98	99.13	-	-	0.98	3.97
417.61	100.00	-	-	0.00	0.00
ALGORITHM I					
10.11	91.88	270.29	36.64	10.1	11.26
20.3	94.37	136.98	25.9	6.85	9.12
40.45	96.29	81.61	23.53	4.76	8.09
60.8	97.2	45.95	17.42	3.6	7.06
86.06	97.86	27.83	13.9	2.49	5.96
96.19	98.12	-	-	2.16	5.37
151.67	98.82	-	-	1.3	4.39
212.11	99.47	-	-	0.77	3.4
414.08	100.00	-	-	0.00	0.00
ALGORITHM II					
10.03	84.94	1567.43	109.62	5.03	7.9
20.06	87.95	1317.92	116.43	4.39	7.57
40.13	91.31	1131.26	138.29	3.92	7.03
60.19	91.48	1122.74	140.63	3.8	7.07
85.27	94.63	515.52	102.89	2.04	6.0
95.3	95.28	478.69	109.17	1.84	5.84
150.47	97.12	332.5	122.24	1.46	5.29
210.66	98.38	-	-	1.39	5.22

As expected, the 2nd algorithm directly accomplishes very good performance in the average error, since the strongest rays (those with fewer interactions with the environment) are processed first. However, those rays that are subject to more interactions, and hence are responsible for the power at the boundaries of the study area, are calculated last. As a result, the topological distribution of intermediate predictions is poor during the entire process. Algorithm I demonstrates the best performance in all statistics. The topological distribution of intermediate predictions is well balanced from the beginning in the entire study area, approaching the k-means result, as expressed by $D_{\rm diff}$ (%). It is impressive to note that 92% coverage is achieved during the initial 2.5% of the total simulation-time ($10''$). Both, the mean and the standard deviation of the error decrease rapidly, and remain lower than the other two approaches. Compared to the SGRI, the proposed method assumed no priority to any source, since even interest was assumed in the entire study area. This was proven a better strategy because each source (either real or secondary) becomes dominant in its illumination zone, and hence its contribution should not be downgraded.

This property is well confirmed in the case examined in Table II. We have assumed a high-priority region, as shown in Fig. 1. We have carefully selected a region that is not directly illuminated by the transmitter, to demonstrate that in such a case the secondary sources could become more important than the real one, and should be assigned a high priority value. All receivers in the study area are weighted, so that $w_{\rm high}/w_{\rm low} = 3$, where $w_{\rm high}$ and $w_{\rm low}$ are the weights of receivers inside and outside of the high-priority region respectively and these weights are properly normalized, so that $\sum_{i=1}^{N} w_i = N$. Apart from the application of weighting in (1)–(5), we similarly weight (6) and apply the same weights for the calculations of the average error. Hence, at a given time t, when estimations have been delivered at M out of N points, the mean error is calculated as $\sum_{i=1}^{M} w_i e_i / \sum_{i=1}^{M} w_i$, where e_i is the difference (in dB) of the intermediate prediction from the final at point i. Note that by applying the proposed algorithms, the real source is in the low priority group, as it does not directly illuminate the high-priority region. The time to perform the illumination test for Algorithms I and II, was 0.43 s, approximately 0.2% of the time to reach the 1 dB average error. Hence, the "illumination-test" time can be safely considered negligible compared to the prediction-time.

The SGRI performance seems downgraded in all categories compared to the previous case. Those samples in the high priority region are now weighted and the corresponding error at these locations now becomes more important. Since the SGRI strategy takes no action to calculate these samples first, its overall performance is worse. The 1st algorithm, on the other hand, succeeds to prioritize the sequence of calculations properly, achieving similar statistics with the non-prioritized case. Once again, it greatly outperforms the SGRI in all categories. It is also important to notice that it achieves a mean error of less than 1 dB and a standard deviation of less than 3.5 dB in approximately half the total simulation time.

The 2nd algorithm suffers from bad topological distribution of intermediate predictions, though it ensures low mean error. Also notice that in both cases, the standard deviation of the 2nd algorithm remains high during the entire process, even when the average error is small (see Table I at 235 s). According to the latter, after each reflection, the reflected ray is stored in a buffer, in order to be processed later. As a consequence, those rays that reach the most distant regions of the study area are processed last. Hence, there are distant samples with large errors that lead to large values of the standard deviation.

Similar behavior is recorded when more regions with the same priority are added in the computations. Algorithm I experiences superior performance, while reaching the same statistics (i.e., specific error value) at similar time intervals. Assuming that the complexity of the scenario is increased, i.e., by considering a greater area with more buildings, the overall prediction time is expected to increase. Hence, the time to deliver intermediate predictions with a given set of requirements is expected to increase accordingly.

V. Conclusion

In this paper, a new criterion to control and prioritize the topological distribution of intermediate predictions in a progressive model has been put forward. The proposed method, combined with the rules already set in [4], provides a solid tool for the inspection of a progressive prediction process.

The criteria presented are also suitable for 3-D predictions. Such predictions could be of interest for outdoor to indoor propagation in a multifloor urban scenario, or for an indoor environment. Especially for the indoor environment, new progressive strategies need to be developed, due to the different propagation-related dominant mechanisms.

We present a new progressive strategy that exhibits superior performance in all statistics both for a prioritized and a non-prioritized case when compared with former approaches. We have shown that each source can become dominant in the region it unobstructively illuminates. The novel thesis introduced is to prioritize sources based on their visibility-relation with the areas of interest and not on the magnitude of the wave that emanates from them.

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