

Particle Filter Object Tracking by a Handheld UHF RFID Reader

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Abstract—A method for target tracking is developed using principles of particle filters. Based on this method we propose the design and functions of a human operated device used to track desired items in a warehouse. More specifically, the items of the warehouse are equipped with *Radio Frequency Identification (RFID)* tags. The device consists of a handheld RFID reader and an antenna. While the human-user moves in the warehouse, the device measures the backscattered phase from the tracked item's RFID tag. Using these measurements and the principles of particle filters, the device guides the user step-by-step, until reaching the tag. The proposed method demonstrates 100% successful identification in the experiments and simulations presented herein.

Index Terms—RFID, particle filter, localization, tracking, phase.

I. INTRODUCTION

RFID technology experiences widespread penetration in the logistics market, particularly in inventorying. Part of the process typically involves personnel, carrying a handheld RFID reader, equipped with a directional antenna. During inventorying, the reader interrogates all tags within range and stores unique ids along with a timestamp in the local database. Usually, it is also desired to locate specific RFID tags during this process. Since, RFID is a non-optical technology, the exact location of the tag is a valuable information which is not provided by any commercial handheld RFID reader.

In this paper, we focus on this problem. We aim to develop a method, that guarantees guidance of the personnel to any specific RFID tag. We assume that the user of the RFID-reader is not a robot, an assumption that raises the difficulty of the problem, since human will not necessarily follow specific instructions. The user continuously receives direction-commands, both orally and on the screen of his RFID reader, until he reaches the target-tag. This work is part of a research project [1].

Prior-art, related to localization of RFID tags, is mainly focused either on fixed installations aiming at localizing all tags within range, or more recently moving robots, aiming at localizing all tags in the surrounding environment of the robot [2]- [8]. Those methods can be further classified, depending on the information that is measured; backscattered power [2]- [4] or phase [5]- [9] of the tag-modulated signal) or the way this

information is handled by each method (e.g. direction finding [9], fingerprinting [3], Synthetic Aperture Radar methods [5]- [8] etc.). To the best of our knowledge, the proposed problem has been treated before only in [10]; i.e. a moving human with a handheld reader, guiding him to a specific RFID tag. In [10], the authors attempted to merge acceleration data from an IMU with the second derivative of phase measurements, collected by a handheld reader, an approach entirely different to what is proposed herein.

The proposed solution applies particle filter theory and updates the filters, based on sequential phase-measurements, taken along successive steps of a human carrying a handheld reader. 100% successful identification is achieved experimentally and by simulation presented next. The proposed solution is expected to be applied in warehouse-management and large retail stores (e.g. imagine RFID-tagged clothes thrown on-top of each other during busy-hours in popular brand-stores could eventually be located).

II. DESCRIPTION OF THE PROBLEM

Consider that a human wishes to locate a specific object, associated with a unique RFID tag. He holds a handheld RFID reader, which will guide him throughout the process, by the following two simple messages:

- "TURN", the user should turn towards the direction shown on the screen of his reader,
- "MOVE", the user should move forward one step.

Each time the user performs a "TURN"/"MOVE" action, he presses a button on the reader, confirming that the action has been completed and receives the new set of instructions, guiding him step-by-step to the object. If the user is unable to move towards the specified direction (e.g. due to an obstacle), he inputs to the reader his actual direction, so that the algorithm updates his new guidance accordingly.

Through the EPC UHF Gen2 protocol, the RFID reader orders only the tracked tag to respond during successive queries. This ensures a very high read-rate (experimentally, we achieved approximately 300 reads per second). The reader collects and stores the EPC, the power (RSSI), the phase and the time of each backscattered signal by the tag.

Sequences of phase-measurements are collected during a "MOVE" action, while similar measurements are collected during a "TURN" actions to check whether the signal from the reader is lost for a specific range of angles. Each set of measurements is initiated as soon as the user presses a

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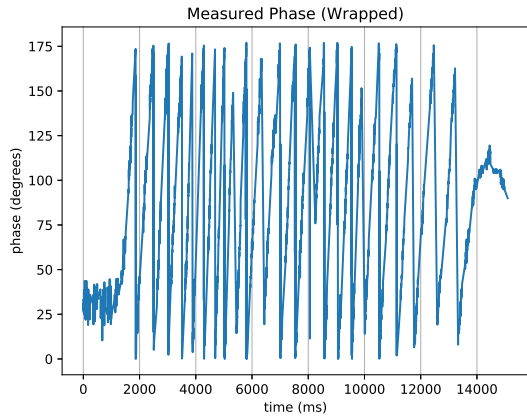


Fig. 1. Sequence of phase measurements

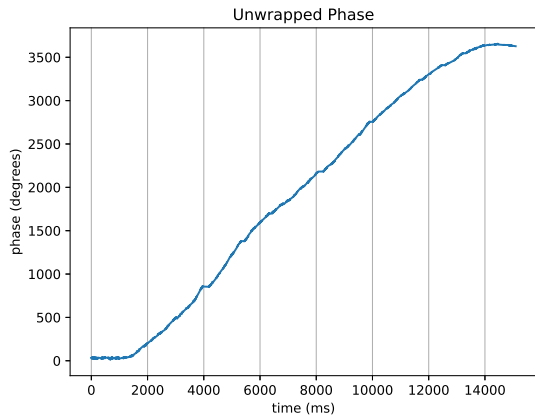


Fig. 2. Unwrapped and smoothed phase measurements

button and completed as soon as the user presses again a button, indicating the specific step has been completed. The proposed algorithm uses the sequence of phase measurements during a "MOVE" step, as demonstrated in Fig. 1. The phase measurements are wrapped in $(0, 2\pi]$ or $(0, \pi]$ intervals, depending on the RFID reader; e.g. the Sargas RFID reader by ThingMagic that we used in the experiments wraps the phase in $(0, \pi]$. Thanks to the high sampling rate (hundreds of reads per second), the measured phase is easily unwrapped, as shown in Fig. 2. The algorithm stores only the phase difference between the last and the first phase-measurement samples, after properly processing the signal to have a smooth curve, detecting outliers. At this point, notice that the slope of the curve actually indicates the direction of the tag (though the problem is not solved in that manner as will be explained next).

The handheld UHF RFID reader's antenna is directional; a desirable effect as tags at the back-lobe of the antenna are not read. In our case, an antenna with a half power beam-width of 68° in the horizontal plane was used.

The process of finding the RFID target is summarized as

follows: the user knows the EPC of the tracked RFID tag. Initially, the reader continuously receives phase measurements from the target. The user searches for a direction in which measurements are received. When such a direction is found, the user initiates the execution of the algorithm. From this point on the user should only turn or move forward, and every such action must be communicated to the reader. Whenever the user moves forward, the reader suggests an angle at which he/she must turn in order to face the RFID target. There may be obstacles in the user's way. In that case, the user turns at another angle, but must inform the reader about this action. Furthermore, the user might find that the tag is undetectable in the "new" direction; then the user rotates to a different direction and informs the reader accordingly.

The proposed algorithm exploits particle filters [11] to guide the user to the target as explained in the following section.

III. PHASE DIFFERENCE MEASUREMENTS

As stated in section II, when the user moves forward the reader collects phase measurements during the movement and calculates the phase difference between the starting and ending point. If $\vec{S} = [s_x, s_y]$ is the starting and $\vec{E} = [e_x, e_y]$ the ending point, then:

$$\vec{E} = \vec{S} + \hat{k}s_{step} + \vec{N}_m \quad (1)$$

Here $\hat{k} = [k_x, k_y] : |\hat{k}| = 1$ is the direction of movement, s_{step} is the distance covered by the user and \vec{N}_m is the uncertainty of the user's movement:

$$\vec{N}_m = [r \cos(\theta_m), r \sin(\theta_m)] \quad (2)$$

r is a random variable of a normal distribution with 0 mean and s_r standard deviation and θ_m is uniformly distributed in $(0, 2\pi]$. \vec{N}_m sizes the uncertainty of the user's movement by a circle of radius r around the expected (or commanded by the reader) ending point \vec{E} .

We measure the phase of the electromagnetic field as it travels from the antenna to the RFID target and returns to the same antenna. In free space, the phase accumulated due to the electromagnetic field propagation is:

$$\phi_{prop} = -\frac{4\pi}{\lambda}d \quad (3)$$

Here d is the distance between the target RFID and the antenna and λ the wavelength of the electromagnetic field. Since there is no coordinate system, each time a tag is tracked, we assume that it is located at the center of our coordinate system, i.e. point $O(0, 0)$. The phase measured on points \vec{S} and \vec{E} is:

$$\phi_S = -\frac{4\pi}{\lambda}|\vec{S}| = -\frac{4\pi}{\lambda}\sqrt{s_x^2 + s_y^2} \quad (4)$$

$$\begin{aligned} \phi_E &= -\frac{4\pi}{\lambda}|\vec{E}| = -\frac{4\pi}{\lambda}\sqrt{e_x^2 + e_y^2} \\ &= -\frac{4\pi}{\lambda}\sqrt{(s_x + k_x s_{step} + r \cos(\theta_m))^2 + (s_y + k_y s_{step} + r \sin(\theta_m))^2} \end{aligned} \quad (5)$$

Actually, there are some common "dc" terms in the above two equations, which are omitted since, we are interested in the phase difference and those terms are subtracted. However, the phase difference $d\phi$ between the starting and ending point, should also include the hardware-measurement error of the RFID reader, denoted as θ_d , i.e.:

$$d\phi = -\frac{4\pi}{\lambda}(|\bar{S}| - |\bar{E}|) + \theta_d \quad (6)$$

θ_d is a normally distributed random variable with 0 mean and s_θ standard deviation. In an effort to approximate the statistics of the expected phase measurement error, we have:

$$|\bar{E}| \approx |\bar{S}| + s_{step}\hat{k} \cdot \frac{\bar{S}}{|\bar{S}|} + \bar{N}_m \cdot \frac{\bar{S}}{|\bar{S}|} \quad (7)$$

From equations (6) and (7):

$$\begin{aligned} d\phi &\approx -\frac{4\pi}{\lambda}(|\bar{S}| - |\bar{S}| - s_{step}\hat{k} \cdot \frac{\bar{S}}{|\bar{S}|} - \bar{N}_m \cdot \frac{\bar{S}}{|\bar{S}|}) + \theta_d \\ &= \left(\frac{4\pi}{\lambda}s_{step}\hat{k} \cdot \frac{\bar{S}}{|\bar{S}|}\right) + \left(\frac{4\pi}{\lambda}\bar{N}_m \cdot \frac{\bar{S}}{|\bar{S}|} + \theta_d\right) \end{aligned} \quad (8)$$

The second part of equation (8) is the noise of the measurement $n_{d\phi}$:

$$\begin{aligned} n_{d\phi} &= \frac{4\pi}{\lambda}\bar{N}_m \cdot \frac{\bar{S}}{|\bar{S}|} + \theta_d \\ &= \frac{4\pi}{\lambda} \frac{s_x r \cos(\theta_m) + s_y r \sin(\theta_m)}{\sqrt{s_x^2 + s_y^2}} + \theta_d \end{aligned} \quad (9)$$

If $E[Q]$ is the mean of the random variable Q , it can be easily shown that the mean and variance of $n_{d\phi}$ are:

$$E[n_{d\phi}] = 0 \quad (10)$$

$$E[(n_{d\phi} - E[n_{d\phi}])^2] = \frac{1}{2} \left(\frac{4\pi}{\lambda}\right)^2 s_r^2 + s_\theta^2 \quad (11)$$

IV. PARTICLE FILTER IMPLEMENTATION

The base of our particle filter implementation is the particles. A particle is a state vector, whose current value depends on its past value and a measurement:

$$p_{t+1} = Ap_t + Bu_t + Cf_t \quad (12)$$

$$u_t = g(m_t) \quad (13)$$

$$m_t = h(p_t, p_{t-1}) + e_t \quad (14)$$

Here:

p_t is the state vector or particle;

u_t is the vector that changes the state vector according to the latest measurements through function $g(m_t)$;

f_t are unmeasured forces or faults;

m_t are the collected measurements;

e_t is the measurement error.

A, B, C are constants that define updating of the state vector according to the (motion) model.

Our goal is to assign a weight w_t^i to each particle p_t^i . This is achieved by using the following algorithm.

A. Particle Filter Algorithm

1) Initialization: A number of random particles $p_0^i, i = 1, 2, \dots, N$ is generated. The initial value of the weight of each particle is $w_0^i = \frac{1}{N}, i = 1, 2, \dots, N$.

2) Weight Update: The weight of each particle is updated depending on the measurements:

$$w_{t+1}^i = w_t^i f_w(m_t^i), i = 1, 2, \dots, N \quad (15)$$

where f_w is a function that affects weighting according to the latest measurements. Finally, the weights are normalized according to $w_{t+1}^i := \frac{w_{t+1}^i}{\sum_{i=1}^N w_{t+1}^i}$.

3) Resampling: A number of random particles is taken with replacement from the set $\{p_t^i\}_{i=1}^N$. Resampling happens only when the following condition is met:

$$\frac{1}{\sum_{i=1}^N (w_{t+1}^i)^2} < N_{th} \quad (16)$$

We have selected $N_{th} = \frac{2N}{3}$.

4) Particle Update: The value of each particle is updated according to:

$$p_{t+1}^i = Ap_t^i + Bu_t^i + Cf_t^i, i = 1, 2, \dots, N \quad (17)$$

5) Return to step 2.

B. Particles

In our case each particle p_t is a three dimension state vector corresponding a point on the plain and a direction (shown in Fig. 3):

$$p_t^i = \begin{bmatrix} x_t^i \\ y_t^i \\ a_t^i \end{bmatrix} \quad (18)$$

Here x_t^i is x-coordinate, y_t^i is the y-coordinate and a_t^i is the angle. Each particle represents a possible position of the user and the direction he/she is facing to. We assume that the desired target is located at the point $O(0,0)$. **All initial particles belong to the same quadrant**; within this quadrant, there is always a location that corresponds to the actual angular relationship between the user and the tag. A visualisation of the algorithm is shown in Fig. 4, where all particles were

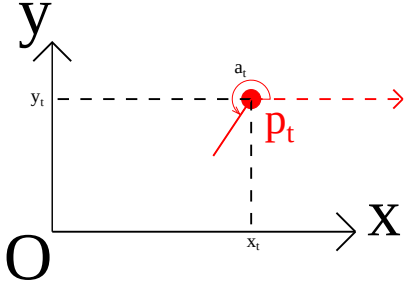


Fig. 3. A representation of a particle (red). Each particle consists of 3 values: the x-coordinate, the y-coordinate and the angle. The angle is measured as if each particle has its own coordinate system.

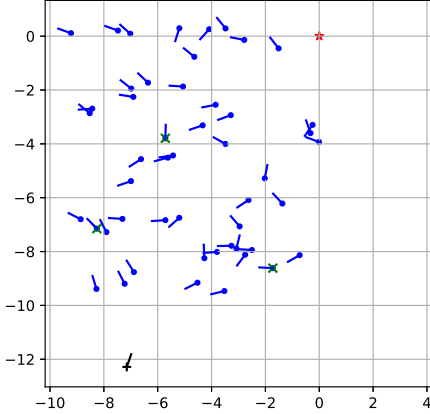


Fig. 4. A representation of the problem. Blue: Current Particles, Red: Target, Green: Important Particles, Black: Real Position and facing direction.

initialized within the left-bottom quadrant. Using the algorithm described in section IV and the already described particle filter implementation, weights are assigned to each particle. The particles with the highest weights are called "Important Particles".

C. Particle Update

In this section it is described how particles are updated at the fourth step of the Particles Filter Algorithm. In section II it is stated that the user must choose between two movement actions when using our method: "TURN" or "MOVE". Whenever the "TURN" action is chosen, the user remains in the same position, but changes the angle he/she is facing to. Whenever the "MOVE" action is chosen, the user moves forward, in the direction he/she is facing to. During that movement phase difference is measured and the simulated particles are updated. Since the user is a human, exact move distance and turn angle cannot be achieved. Noise is included in our model to simulate this uncertainty.

If J is the number of "TURN" actions between two consecutive "MOVE" actions and $a_d^j, j = 1, 2, \dots, J$ are the turn angles of these "TURN" actions, then:

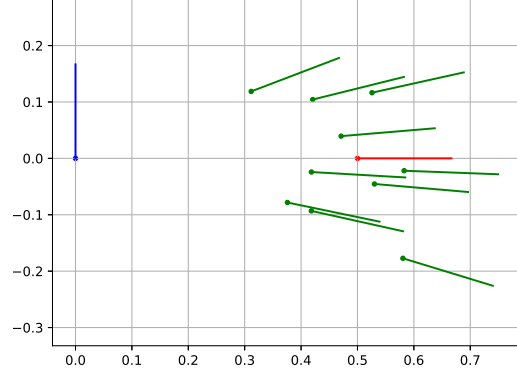


Fig. 5. An example of possible outcomes of the particle update. The blue particle is the original, the red is the updated in the noiseless scenario and the green are the resulted updated particles of a simulation. Here $s_{step} = 0.5$, $s_r = 0.1$, $a_u = -\pi/2$ and $s_a = 0.2$.

$$a_u = \sum_{j=1}^J a_d^j \quad (19)$$

The particles are updated according to:

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ a_{t+1} \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \\ a_t \end{bmatrix} + \begin{bmatrix} (s_{step} + r)\cos(a_t + a_u + n_a) \\ (s_{step} + r)\sin(a_t + a_u + n_a) \\ a_u + n_a \end{bmatrix} \quad (20)$$

s_{step} is the distance covered by the user, r is a random variable of a normal distribution with 0 mean and s_r standard deviation. a_u is the total turn angle since the last particle update and n_a indicates the uncertainty of the real turn angle. It is a normally distributed variable with 0 mean and s_a standard deviation. An example of possible outcomes of the particle update are shown in Fig. 5.

Finally *jittering* was used after the particle update [12]:

$$p_t := \begin{bmatrix} x_t \\ y_t \\ a_t \end{bmatrix} + \begin{bmatrix} n_j^x \\ n_j^y \\ 0 \end{bmatrix} \quad (21)$$

n_j^x and n_j^y are normally distributed random variables with 0 mean and $2s_r$ standard deviation.

V. WEIGHT CALCULATION

In this section it is described how the weight of each particle is calculated. Let $d\phi_{t+1}$ be the measured phase difference between times t and $t+1$. To each particle i a measurement m_{t+1}^i is assigned according to:

$$\begin{aligned} m_{t+1}^i &= h(p_{t+1}^i, p_t^i) + e_{t+1} \\ m_{t+1}^i &= \frac{-4\pi}{\lambda} \left(\sqrt{(x_t^i)^2 + (y_t^i)^2} - \sqrt{(x_{t+1}^i)^2 + (y_{t+1}^i)^2} \right) \\ &\quad - d\phi_{t+1} \end{aligned} \quad (22)$$

Equation (22) shows that m_{t+1}^i is the difference between the measured phase difference and the phase difference that would be measured if the user's position and facing direction were the same as this of particle i . An approximation of the statistics of the error e_{t+1} of (22) was given in (8) and (9). Once m_{t+1}^i is calculated, the weight of the particle is updated according to equation (15). In our case $f_w(m_{t+1}^i) = \left(\frac{1}{m_{t+1}^i}\right)^2$ is used.

A. Suggested Turn Angle

When the user completes a movement, the reader suggest a turn angle a_{sug} in order for the user to be facing the RFID target. In this section it is explained how this angle is calculated. Every particle has a weight assigned to it. The higher the weight, the more similar its behavior to that of the user, in terms of measured phase difference. It is assumed that the RFID target is at the point $O(0,0)$. The change of the angle da_t^i of each particle so that it is facing the point $O(0,0)$ is calculated. a_{sug} is the weighted sum of these changes of angle.

For the particle p_t^i , da_t^i is:

$$da_t^i = \arctan\left(\frac{-y_t^i}{-x_t^i}\right) - a_t^i \quad (23)$$

For the calculation of $\arctan()$ we consider in which quadrant the point $(-x_t^i, -y_t^i)$ is. The suggested turn angle a_{sug} is:

$$a_{sug} = \sum_{i=1}^N w_t^i da_t^i \quad (24)$$

VI. SIMULATIONS

In this section simulations of the problem are presented. The RFID target is assumed to be on the point $O(0,0)$. The starting position and the direction of the user is random. The distance between the users and the target is 10 meters. The half power beam width of the supposed antenna is 180° , exaggerating the read-range of the RFID antenna, thus allowing for greater mistakes in the direction-guidance process. Furthermore, big mistakes in the actual behavior of the user (rotation and step) are introduced below.

s_{step} , s_r and s_a depend on the movement of the human user, where s_{step} is the desired step value, ordered by the RFID reader and the other two variables size the user's errors in performing what was instructed. Their values were chosen to fit a worst-case scenario: $s_{step} = 0.5m$, $s_r = 0.1m$ and $s_a = 11.5^\circ$. The value of s_r implies that the user's step ranges between 0.4m to 0.6m for 67% of the time and between 0.2m to 0.7m for 95% of the time. Similarly, the range of direction-error taken by the user is very big. Such big errors are considered excessive for the trained personnel (i.e. requesting one to move by 50cm and he actually moves by 20cm), performing inventorying; however such simulations would ensure that the method performs well, under extreme errors taken by human-personnel. s_θ depends on the RFID

TABLE I
SIMULATION RESULTS.

N	"STEPS"	"TURNS"	"DISTANCE"	"TIME (ms)"
10	30.0	35.9	15.4	10.4
20	29.4	35.0	15.0	11.8
50	29.2	34.5	15.0	17.8
100	28.5	33.2	14.6	29.2
300	28.3	33.0	14.5	90.7
1000	28.0	32.2	14.3	473.9

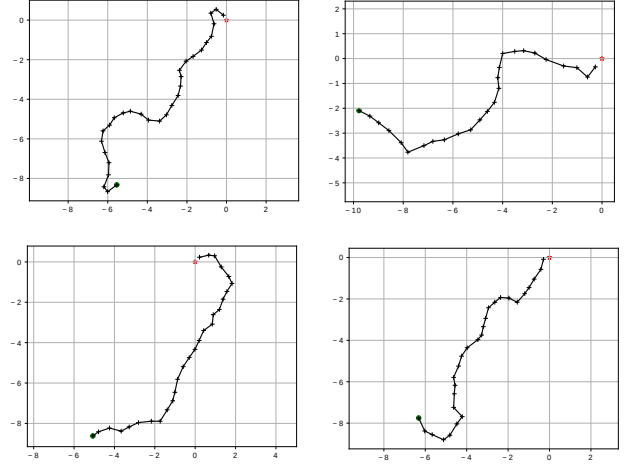


Fig. 6. Example of the route the user followed in simulations. The black "+" are the positions of the user, the green "o" is the starting position of the user and the red "*" is the position of the RFID target. All dimensions are in meters.

reader. We set $s_\theta = 5.5^\circ$, based on the variability of measured data at a fixed set-up.

The simulations are evaluated by three measures: "STEPS", "TURNS" and "DISTANCE". "STEPS"/"TURNS" is the number of steps/turns the user made until the RFID target was found. "DISTANCE" is the distance (m) the user covered. It should be emphasized that in **all** simulations, the user **successfully** reached the target RFID tag. Increasing the number of particles of the simulation leads to decreased values of all three measures at the expense of longer update-times between successive steps, which is shown at the last column of Table I. We used 10, 20, 50, 100, 300, and 1000 particles. Even when 1000 particles were used, the time needed to calculate the suggested angle was less than 1 second. This suggests that our method can be used for real time applications. The results of the simulations are shown in Table I. Characteristic examples of the route the user followed in simulations are shown in Fig. 6.

VII. EXPERIMENTS

We have performed several experiments in an office environment, using the Sargas UHF RFID Reader by THINGMAGIC and The SlimLine A5010 microstrip antenna by Times-7. In all cases, we started from a 3m distance to the tag. Initially, the reader was rotated, until the tag was read. Then a single



Fig. 7. A photograph of the resulted route of an experiment. The route is represented by the white tape on the floor. The target is the box on the chair.

step is taken by the user, the unwrapped phase difference is measured and the iterative process begins. The particles are initiated and weighting is applied. A suggested turn angle is given by the reader and the user rotates and takes the next step. When the user is requested to move towards an obstacle, a different rotation angle is taken and the reader is informed accordingly, applying (19). In all cases, we were able to track the RFID tag in a few steps, corresponding to just a few seconds; as the actual step and angle errors of the moving user were much smaller than the simulated values. In some cases, the reader-suggested rotation angle faced an obstacle and the user took a different direction and updated the system accordingly. Representative results of the paths followed during the measurements are shown in Figs. 7 and 8.

VIII. CONCLUSIONS

In this paper, we have presented a prototype method for RFID tag tracking by a human, carrying a handheld reader. The method exploits particle filter theory and sequences of phase measurements collected during the process. The method has been verified by simulation and experiments conducted in an office environment. 100% success was recorded. Even though the proposed problem targets standard inventorying, it could also be applied in assisted living, where visually impaired people could be assisted to locate items (e.g. medication) inside their environment, where the visual-instructions GUI of the reader should be enhanced with oral orders.

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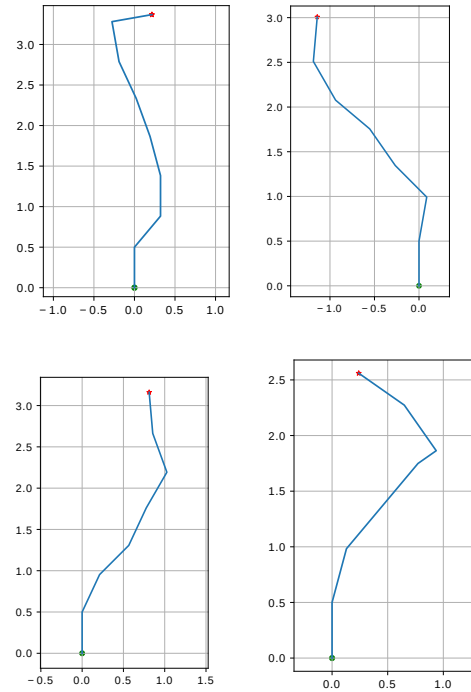


Fig. 8. Representation of routes resulted by experiments. The blue line is the route, the green point is the starting point, and the red point is the ending point. The distance of the RFID target and the ending point is less than 20cm. All axes dimensions are in meters.

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