Discrimination of regimes of dynamical systems using measures of correlation

A. Papana, D. Kugiumtzis
General Department, Faculty of Technology, Aristotle University of Thessaloniki.

Abstract

Linear and nonlinear measures of correlation are compared in distinguishing different regimes of a dynamical system on the basis of time series observation. The linear measures are the autocorrelation r(t) for certain delays and the sum of autocorrelations (Portmanuette) until a maximum delay, Q(t). Similarly, as nonlinear measures we consider the mutual information I(t) on the same delays and the sum of I(t) for the same maximum delay, M(t). We also examine whether the discrimination of the dynamical regimes can be achieved equally well, or even better, using as measures the p-values of the surrogate data test for nonlinearity (where the test statistics are the nonlinear measures). In this way, the p-values can be considered as an indirect measure of departure from linear correlations.

To assess the three correlation measure types, we make Monte Carlo simulations on well-known linear and nonlinear systems varying their complexity by monitoring control parameters of the system (AR model, Mackey Glass). We also apply the correlation measures to multi-channel EEG from epileptic patients. For each record and channel we create time series of the same length from successively overlapping segments and calculate the measures from these time series. We check if there is discrimination between EEG record from late preictal state (1/2 minutes before seizure onset) and from early preictal state (some hours before seizure onset), where the patient is considered to have a more regular cerebral operation.

<table>
<thead>
<tr>
<th>Measures of linear correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation r(t), for t = 1, 5, 10, 20, 30, 60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measures of nonlinear correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mutual information I(t), for t = 1, 5, 10, 20, 30</td>
</tr>
</tbody>
</table>

Methods

For the simulated data, the discrimination of two different states of the system was evaluated with ROC curves (Receiver Operating Characteristic Curves). Naming one state as “positive” and the other “negative”, ROC makes a graphical representation of the trade-off between the sensitivity (true positive rate) and specificity (false negative rate). The plot shows the sensitivity on the Y axis and (1 - specificity) on the Y axis. The curve always goes from point (0,0) to (1,1) a threshold cut-off (0,0) is where the classifier finds no positives (all values are classified to state “positive”), (1,1) where everything is classified as positive (all values are classified to state “negative”). Thus for each cut-off the cumulative function of the distributions regarding each state is calculated. The area under the ROC is a convenient way of comparing classifiers. A random classifier has an area of 0.5 (no distinction of the two states for any cut-off value), while an ideal classifier has an area of 1 (absolute distinction for the whole range of cut-off values).

For EEG data the discrimination between pre-ictal & pre-ictal state was evaluated using comparison tests (for ROC, distributions could not be estimated reliably due to small sample sizes).

Results from simulations

Discrimination between quadratic and cubic transform of time series generated by AR(9) (n=1000) for the measures: r(t), I(10), P(40), M(40), P(40).

Results from data

Discrimination of Mackey Glass with s=70 & 100, n=1000 with 50% noise level.

Conclusions

From the simulations we can see that the cumulative measures (Q(40)) and M(40) outstand in performance, while p-values turned out to have poor discriminative power. Correlation measures can discriminate between pre-ictal and pre-ictal states, especially when examining pre-ictal records from many hours before seizure onset. The two discrimination tests (t-test and Wilcoxon rank sum test) for means and medians have no significant differences.

References