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Routes to Chaos in Resonant Extrasolar Planetary Systems

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Summary. We study the factors that affect the stability and the long term evolution of a *resonant* planetary system. For the *same* resonance, the long term evolution of a resonant planetary system depends on the the relative orientation of the planetary orbits, on the phase of the planets on their orbits and on the proximity to a *periodic* resonant planetary system, either stable or unstable. Chaotic property is not always associated with a disruption of the system and the system may remain bounded for a long time.

Key words: Periodic orbits, resonance, extrasolar systems, routes to chaos.

1.1 Introduction

During the past 15 years many planetary systems were observed, called *extrasolar planetary systems*. Some of these systems are very different from our own Solar System, because they have massive planets close to the central star, which we will call the *sun*, of the order of the mass of Jupiter or larger, and large planetary eccentricities. In some of these planetary systems two or more planets were observed. In the case of two planets close to each other, the two planets are in mean motion resonance. Examples are HD 82943 and GLIESE 876, at the 2:1 resonance and 55Cnc at the 3:1 resonance. There are several problems associated with the study of the extrasolar systems, as their formation, their evolution and their dynamical stability. In this paper we will study some aspects of the dynamical stability of extrasolar planetary systems.

There are different approaches to the study of the dynamical evolution of a planetary system and on the mechanisms that stabilize the system, or generate chaotic motion and instability: Beaugé and Michtchenko 2003, Beaugé et al. 2003, Gozdjiewski et al. 2003, Michtchenko et al. 2006, Malhotra 2002, Lee 2004, Dvorak et al. 2005. In these papers different methods have been applied, as the averaging method, direct numerical integrations of orbits, or various numerical methods which provide indicators for the exponential separation of nearby orbits. In this way the regions where stable motion exists have been detected, in the orbital elements space. A special problem in the study of the extrasolar planetary systems is the existence of large planetary eccentricities. It is believed that the observed planetary systems were not created in their present configuration, but came to their observed, evidently *stable*, state with large eccentricities, following a *migration* process, as the protoplanetary disc in which they were formed slowly dissolved (Ferraz-Mello et al. 2003, Beaugé et al. 2006). It is obvious that the planetary system was trapped, at the end of the migration process, to a stable configuration.

We will study the regions of the phase space where we can have stable, bounded, motion of a planetary system, and in particular stable *resonant* motion. This requires the knowledge of the topology of the phase space. The topology of the phase space in any dynamical system is determined critically by the periodic orbits, which are the "backbone" of the phase space, although they are a set of measure zero. Close to a stable periodic orbit we have stable librations and the motion in phase space takes place on a torus. On the contrary, close to an unstable periodic orbit we may have irregular, chaotic, motion and in many cases the system disrupts into a binary system (the star and one planet) and an escaping planet. We study the evolution from order to chaos as the perturbation on a stable resonant periodic orbit increases. Since an arbitrary resonant planetary system is not always stable, we study the factors that affect the stability of a planetary system at a *fixed* resonance.

1.2 Resonant extrasolar planetary systems

We consider planetary systems with two planets close to each other, moving in the same plane. Since the gravitational interaction between the planets cannot be ignored, even for very small planetary masses, the model we use is the *general three body problem*, for planar motion. The three bodies are the sun, S, and the two planets, P_1 and P_2. It can be proved (Hadjidemetriou 1975) that families of periodic orbits in the planar general three body problem exist, in a *rotating* frame xOy, whose x-axis is the line $S-P_1$, with origin at the center of mass of these two bodies, where S is the Sun and P_1 the inner planet. This implies that the *relative* configuration is repeated in the inertial frame. We assume that the center of mass of the whole system is at rest with respect to an inertial frame. We have four degrees of freedom, for planar motion, with generalized variables x_1, x_2, y_2, θ (see Figure 1a). This is a non uniformly rotating frame, where the first planet, P_1, moves on the x-axis and the second planet, P_2, moves in the xy plane. It turns out that the angle θ is ignorable, so the degrees of freedom are reduced to three and the study of the system can be made in the rotating frame only, with variables x_1, x_2, y_2. Note that the phase space in the rotating frame is six dimensional. The dimensions of the phase space can be reduced further by considering the *Poincaré map* on a surface of section. By the Poincaré map we reduce the dimensions of the phase space, without losing the generality of the problem, and in addition we eliminate the unnecessary details that are not important in the study of the long term evolution of the system. In the present study, in all our computations, we consider the surface of section

$$y_2 = 0 \ \ (\dot{y}_2 > 0), \ \ H = h = \text{constant}.$$

The phase space of the Poincaré map is the four dimensional space x_1, $\dot{x}_1$, x_2, $\dot{x}_2$ ($y_2 = 0$ and $\dot{y}_2$ is obtained from $H = h$, $\dot{y}_2 > 0$) and the periodic orbits are represented as *fixed points* of the map.

There are two types of periodic planetary orbits in the general planar three body problem. These are *non resonant* periodic orbits with nearly circular orbits of the two planets and *resonant* periodic orbits, with nearly elliptic orbits of the two planets. In this latter case the two planets are in *mean motion resonance*. All these orbits belong to *monoparametric families of periodic orbits*. In particular, along the families of the resonant elliptic orbits, the resonance (ratio T_2/T_1 of the planetary periods in the inertial frame) is almost constant, but the planetary eccentricities increase, starting from zero values, and may reach high values. There are resonant families for all values of the resonance T_2/T_1. Some of these families (or parts of them) are stable. The elliptic orbits may be symmetric or asymmetric. In the symmetric orbits, that we shall mainly study, the lines of apsides of the two planetary orbits coincide and the perihelia may be alighned ($\Delta\omega = 0^0$) or antialigned ($\Delta\omega = 180^0$), while the planets are, at $t = 0$, at their perihelia or aphelia. We remark that the above mentioned nearly circular or elliptic planetary orbits refer to the inertial frame, but the motion is *exactly periodic* in the rotating frame, where the shape of the orbit may be quite different (see, for example, Figures 1.2b, 1.9b).

As we mentioned, all the periodic orbits with elliptic planetary orbits are resonant. The periodic orbits appear as *fixed points* on the Poincaré map and it is these fixed points that determine the topology of the phase space. It is close to the stable, resonant, fixed points that a planetary system can be trapped. This makes clear the importance of the resonances in the dynamical study of the extrasolar planetary systems, and explains why there are many observed resonant planetary systems. A systematic study of the dynamics of planetary systems along these lines is presented in Hadjidemetriou 2006. Works on the dynamics of planetary systems based on periodic orbits are in Hadjidemetriou 1976, 2002, Psychoyos and Hadjidemetriou 2005 and Voyatzis and Hadjidemetriou 2005, 2006.

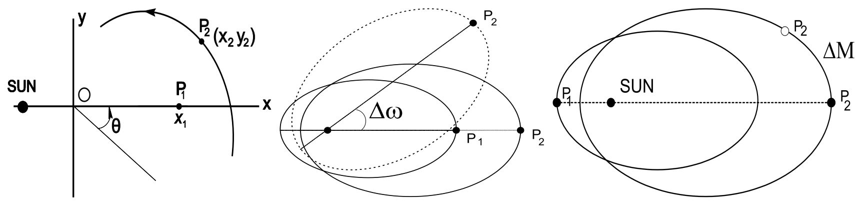

Fig. 1.1. (a) The rotating frame xOy and the coordinates x_1, x_2, y_2 for the position of the three bodies in the rotating frame and θ for the orientation of the rotating frame. (b) The perturbation by rotating the orbit of P_2 by $\Delta\omega$. (c) The perturbation by shifting the planet P_2 on its orbit by ΔM, where M is the mean anomaly.

We focus our attention on the stability in the neighborhood of a resonant orbit. There are two types of resonance: The resonant *periodic* orbits, mentioned above, which we shall call *exact resonance*, and the orbits where the semimajor axes of the two planets correspond to a certain resonance, but the eccentricities and the orientation of the planetary orbits are different from those of the exact periodic motion. We start from an exact symmetric resonant periodic orbit and we consider two types of initial perturbation, by destroying the symmetry: (1) We rotate the orbit of the outer planet P_2 (in the inertial frame) by an angle $\Delta\omega$, keeping all other elements fixed and the two planets at their perihelia or aphelia, accordingly (Figure 1.1b). (2) We keep the position of the two planetary orbits in their symmetric configuration, but now we shift, at $t = 0$, the second planet P_2 on its orbit by an angle ΔM (Figure 1.1c). In both cases the resonance is kept fixed. Thus, by increasing the perturbation $\Delta\omega$, or ΔM, we study the evolution from order to chaos, for *the same resonance.* Close to a *stable* periodic motion we expect a libration of the orbital elements about their exact resonant values, and in particular a libration of the angle $(\omega_2 - \omega_1)$ between the line of apsides of the two planetary orbits. But what happens for larger perturbations? Or what behavior should we expect in the vicinity of an *unstable* periodic orbit? We present four typical examples of resonant periodic orbits, with different behavior as far as their evolution is concerned, as a perturbation increases. Two at the 2:1 resonance, both stable, and two in the 3:1 resonance, one stable and one unstable. As we will see in the following, the resonance alone is not enough to stabilize a planetary system. There are many factors that affect the stability of a resonant system, and mainly the phase and the deviation from symmetry. We study the different routes to chaos, as the perturbation increases, and also the behavior of the angle $(\omega_2 - \omega_1)$, from libration, to rotation, to chaos, or to chaotic interchange between libration and rotation, as the perturbation increases.

1.3 A stable system at the 2:1 resonance. $(\omega_2 - \omega_1) = 0^0$.

We consider a 2:1 resonant stable periodic orbit, corresponding to a planetary system with masses $m_{SUN} = 0.9978$, $m_1 = 0.0008$, and $m_2 = 0.0014$ (normalized to $m_{SUN} + m_1 + m_2 = 1$), semimajor axes $a_1 = 0.884$ AU, $a_2 = 1.415$ AU ($T_2/T_1 = 2.024$) and eccentricities $e_1 = 0.774$, $e_2 = 0.394$. The orbit is symmetric, with the lines of apsides of the two planets aligned, $(\omega_2 - \omega_1) = 0^0$ and the two planets are at perihelion at $t = 0$ (see Figure 1.2). This orbit belongs to a family of stable periodic orbits for the masses of the extrasolar planetary system HD82943 (Hadjidemetriou 2002).

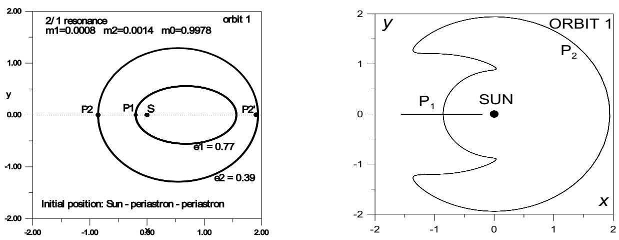

Fig. 1.2. (a) The 2:1 resonant periodic orbit with the two planetary orbits aligned, $\omega_2 - \omega_1 = 0^0$, in the inertial frame. The motion is not periodic in this frame. (b) The same periodic orbit in the rotating frame, where it is exactly periodic.

We perturb the periodic motion by rotating the orbit of the second planet by an angle $\Delta\omega$. The semimajor axes and the eccentricities of the two planets are fixed, and also the planets are at their perihelia at $t = 0$. In this way, we destroy the symmetry, but the resonance is always the same, equal to 2:1. We start with a small angle of rotation $\Delta\omega = 15^0$ (Figure 1.3) and increase the perturbation to $\Delta\omega = 20^0$ (Figure 1.4), $\Delta\omega = 25^0$ (Figure 1.5), $\Delta\omega = 27^0$ (Figure 1.6) and $\Delta\omega = 30^0$ (Figure 1.7). In

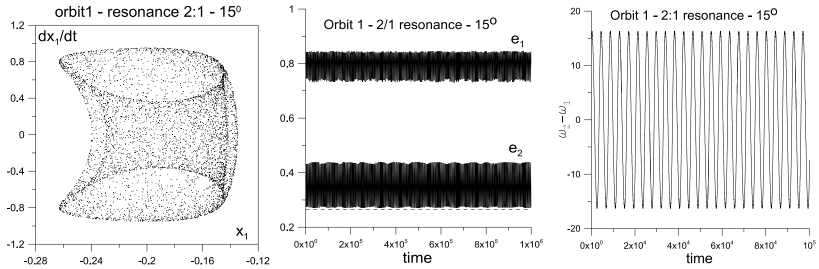

Fig. 1.3. The evolution of the orbit of Figure 1.2, corresponding to the perturbation $\Delta\omega = 15^0$. (a) The Poincaré map (projection on the $x_1\ \dot{x}_1$ plane). The motion is near a torus. (b) The evolution of the eccentricities, which librate around the values of the periodic motion. (c) The libration of $(\omega_2 - \omega_1)$ around 0^0.

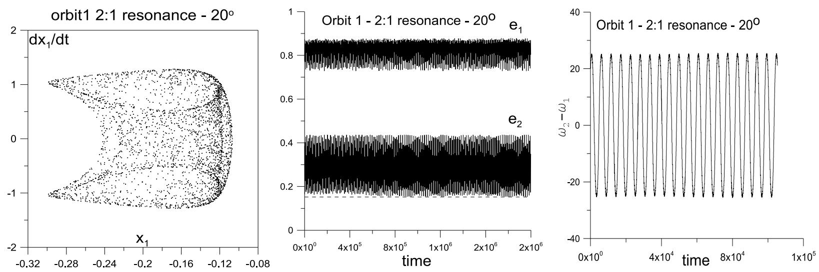

Fig. 1.4. The same as Figure 1.3, $\Delta\omega = 20^0$. The angle $(\omega_2 - \omega_1)$ librates around 0^0.

these Figures we present the Poincaré map (a projection on a coordinate plane, as indicated), the evolution of the eccentricities and the evolution of the angle $(\omega_2 - \omega_1)$ between the line of apsides. We note that for a small value of the perturbation, $0^0 < \Delta\omega < 25^0$, the Poincaré map is on a well defined torus and the eccentricities librate around their original values (corresponding to the exact periodic motion). The angle $(\omega_2 - \omega_1)$ *librates* around 0^0 with an amplitude that increases an the perturbation increases. When the perturbation increases to $\Delta\omega = 27^0$, the motion is still ordered, but now the angle $(\omega_2 - \omega_1)$ *rotates*. For a still larger perturbation, $\Delta\omega \geq 30^0$, the motion becomes chaotic, and eventually the system disrupts to a binary system (the sun and P_2) while the planet P_1 escapes. In all bounded cases

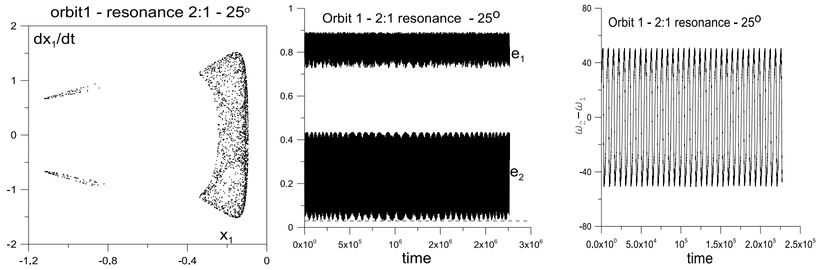

Fig. 1.5. The same as Figure 1.3, $\Delta\omega = 25^0$. The angle $(\omega_2 - \omega_1)$ librates around 0^0.

the resonance T_2/T_1 remains very close to the 2:1 value. A global view of the

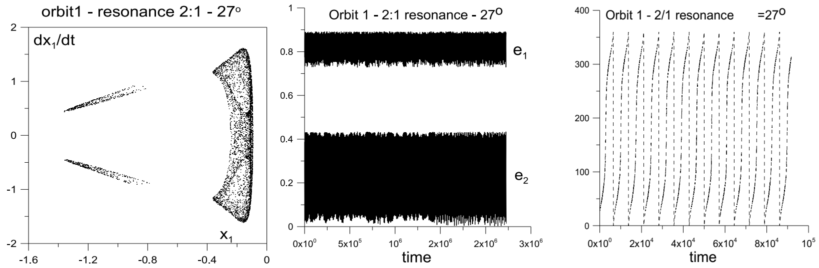

Fig. 1.6. The same as Figure 1.3, $\Delta\omega = 27^0$. The angle $(\omega_2 - \omega_1)$ rotates.

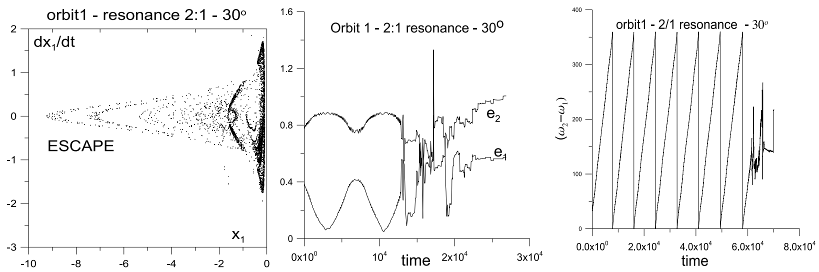

Fig. 1.7. The evolution of the orbit of Figure 1.2, corresponding to the perturbation $\Delta\omega = 30^0$. The evolution is strongly chaotic. (a) The Poincaré map (projection on the $x_1\, \dot{x}_1$ plane). The planet P_1 escapes. (b) The evolution of the eccentricities. (c) The evolution of $(\omega_2 - \omega_1)$.

behavior presented above with some typical cases, is shown in Figure 1.8a, where

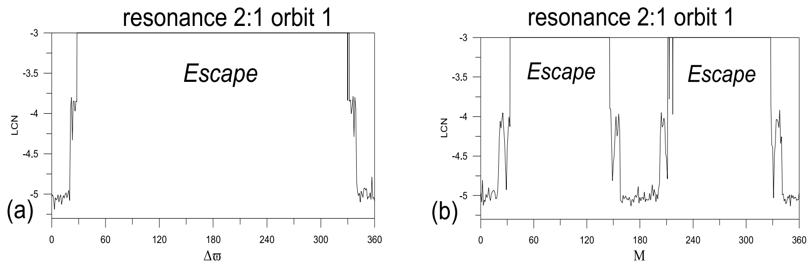

Fig. 1.8. A global view of the stable and unstable regions by the method of LCN: (a) Perturbation in $\Delta\omega$. (b) Perturbation in ΔM.

we present the Lyapunox Characteristic Numbers, LCN, (Contopoulos and Voglis 1998) for all values of $\Delta\omega$.

In Figure 1.8b we present a global view of the behavior of the resonant system when the perturbation is the shifting of the outer planet P_2 on its orbit by ΔM (Figure 1.1c). The results are similar to those for the $\Delta\omega$ perturbation. We note that there is another stable window at $\Delta M = 180^0$, but this is not different from the stable window at $\Delta M = 0^0$. This is so because, due to the 2:1 resonance, after a time interval equal to half the period, the planet P_1 is again at perihelion, but the planet P_2 is at aphelion, $(\omega_2 - \omega_1) = 180^0$. So, these two stable windows represent the same configuration.

1.4 A stable system at the 2:1 resonance. $\omega_2 - \omega_1 = 180^0$.

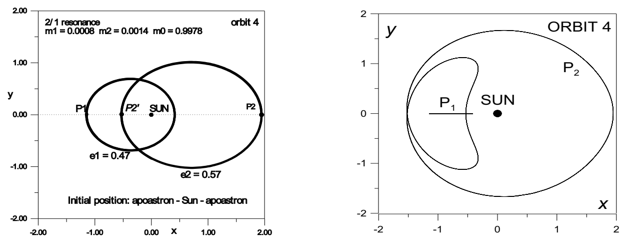

Fig. 1.9. (a) The 2:1 resonant periodic orbit with the two planetary orbits antialigned, $(\omega_2 - \omega_1) = 180^0$, in the inertial frame. In this frame it is not periodic. (b) The same orbit in the rotating frame, where it is exactly periodic.

In this section we consider another 2:1 resonant stable periodic orbit, corresponding to a planetary system with masses $m_{SUN} = 0.9978$, $m_1 = 0.0008$, and $m_2 = 0.0014$, semimajor axes $a_1 = 0.782$AU, $a_2 = 1.238$AU ($T_2/T_1 = 1.992$) and eccentricities $e_1 = 0.471$, $e_2 = 0.573$. The orbit is symmetric, with the lines of apsides of the two planets antialigned, $(\omega_2 - \omega_1) = 180^0$, and the two planets are at aphelion

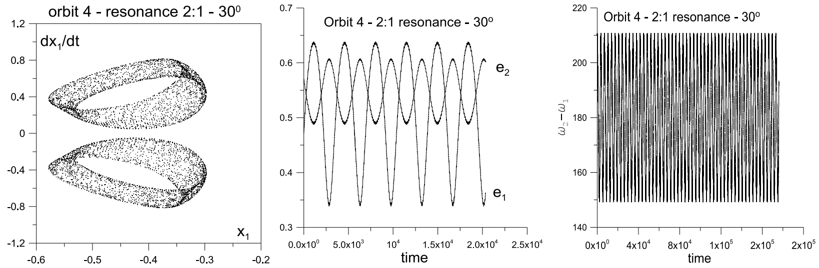

Fig. 1.10. The evolution of the orbit of Figure 1.9, corresponding to the perturbation $\Delta\omega = 30^0$. (a) The Poincaré map (projection on the $x_1\ \dot{x}_1$ plane). The motion is on a torus. (b) The evolution of the eccentricities, which librate around the values of the periodic motion, with large ampltude. (c) The libration of $(\omega_2 - \omega_1)$ around 180^0.

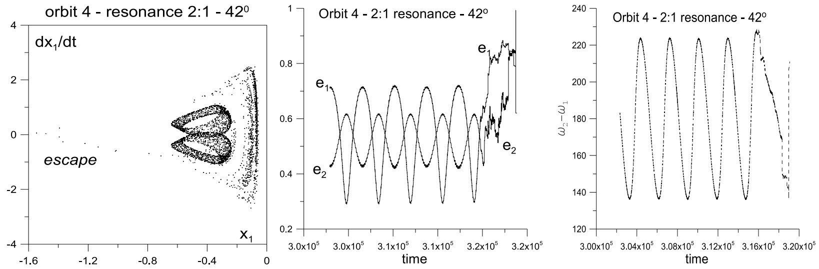

Fig. 1.11. The evolution of the orbit of Figure 1.9, corresponding to the perturbation $\Delta\omega = 42^0$. (a) The Poincaré map (projection on the $x_1\ \dot{x}_1$ plane). The motion is chaotic, and the planet P_1 escapes. (b) The evolution of the eccentricities, which is chaotic, after a libration for a long time. (c) The evolution of $(\omega_2 - \omega_1)$ which is chaotic, after a libration for a long time. In (b) and (c) only the end part of the evolution is shown, after $t > 3 \times 10^5$

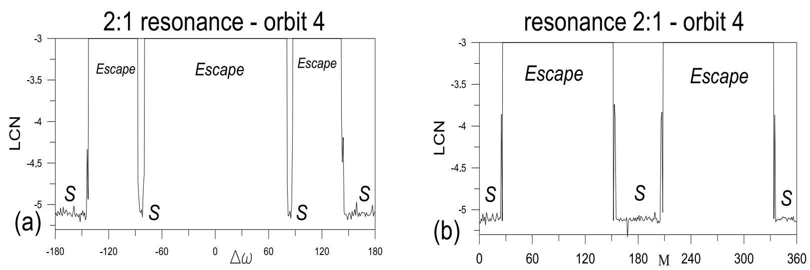

Fig. 1.12. A global view of the stable and unstable regions by the method of LCN: (a) Perturbation in $\Delta\omega$. (b) Perturbation in ΔM.

at $t = 0$, (see Figure 1.9a) and belongs to a family of stable periodic orbits (Hadjidemetriou 2002). We work as in the previous case, and the results are presented in Figures 1.10 and 1.11. We note that for $\Delta\omega < 40^0$ the motion is ordered and the angle $(\omega_2 - \omega_1)$ librates around 180^0. For a perturbation $\Delta\omega \geq 42^0$ the motion becomes chaotic, after a long time of motion on a torus. A global view, by making use of LCN is given in Figure 1.12a. In this Figure we note that there exists another stable window at $\Delta\omega \approx \pm 90^0$. This corresponds to a stable family of non symmetric periodic orbits at the 2:1 resonance (Voyatzis and Hadjidemetriou, 2005). In Figure 1.12b we present a global view of the stable regions when the perturbation is ΔM. The behavior is similar to the $\Delta\omega$ perturbation.

1.5 An unstable system at the 3:1 resonance. $(\omega_2 - \omega_1) = 0^0$

In this section we consider a 3:1 resonant *unstable* periodic orbit with small eccentricities, corresponding to a planetary system with masses $m_{SUN} = 0.99903$, $m_1 = 0.00078$, and $m_2 = 0.00019$, semimajor axes $a_1 = 1.035$ AU, $a_2 = 2.163$ AU ($T_2/T_1 = 3.019$) and eccentricities $e_1 = 0.051$, $e_2 = 0.074$. The orbit is symmet-

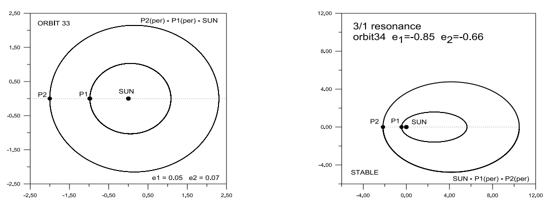

Fig. 1.13. Two periodic orbits at the 3:1 resonance, at $\omega_2 - \omega_1 = 0^0$: (a) An unstable orbit. (b) A stable orbit. These orbits are not exactly periodic in this frame.

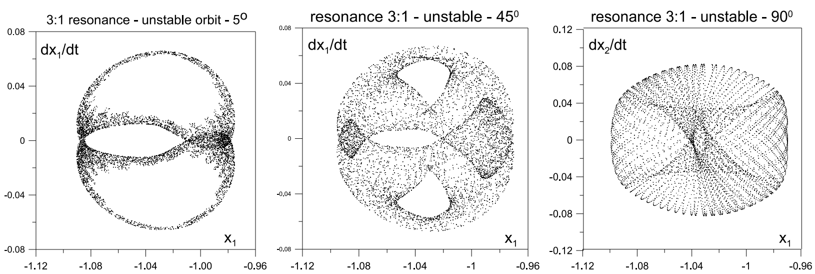

Fig. 1.14. The Poincaré map at the unstable periodic orbit of Figure 1.13a (projection on a plane, as indicated). (a) Perturbation $\Delta\omega = 5^0$. The Poincaré map is bounded, but its boundary is a fractal. (b) Perturbation $\Delta\omega = 45^0$. The Poincaré map is on a torus. (c) Perturbation $\Delta\omega = 90^0$. The Poincaré map is on a torus.

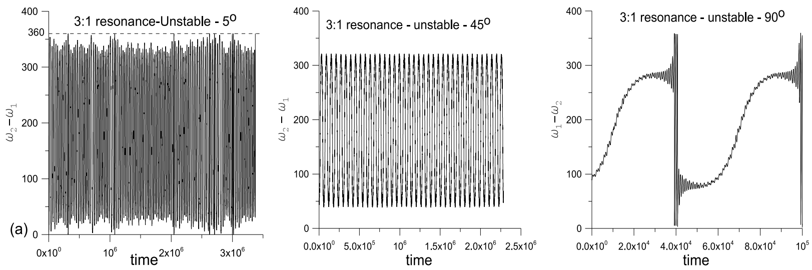

Fig. 1.15. The evolution of the angle $(\omega_2 - \omega_1)$ at the unstable periodic orbit of Figure 1.13a. (a) Perturbation $\Delta\omega = 5^0$. There is a chaotic transition between libration with large amplitude and rotation. (b) Perturbation $\Delta\omega = 45^0$. The motion is ordered and the angle $(\omega_2 - \omega_1)$ librates. (c) Perturbation $\Delta\omega = 90^0$. A slow increase of $(\omega_2 - \omega_1)$ for a long time followed by a rotation for a short time, at regular intervals.

ric, with the lines of apsides of the two planets aligned, $(\omega_2 - \omega_1) = 0^0$, and the two planets are at perihelion at $t = 0$, (see Figure 1.13a) and belongs to a family of unstable periodic orbits (Voyatzis and Hadjidemetriou 2006) for the masses of 55Cnc. We work as in the previous case, and the results are presented in Figures 1.14 and 1.15. We start with a small perturbation $\Delta\omega = 5^0$, close to the unstable periodic orbit (Figure 1.14a) and we note that the motion is bounded, as shown by the Poincaré map. (Computations for a much longer time than presented in this Figure revealed that the motion is indeed bounded). A further increase of the perturbation, to $\Delta\omega = 45^0$ (Figure 1.14b) and to $\Delta\omega = 90^0$ (Figure 1.14c) shows that the motion is bounded. The difference however of these latter two cases from the perturbation close to the unstable periodic orbit is that the outer boundary of the Poincaré map close to the unstable periodic orbit seems to be a *fractal*, while the perturbed orbits far from the unstable periodic orbit are well defined *tori*. In Figures 1.15a,b,c we present the evolution of the angle $(\omega_2 - \omega_1)$ for the above three perturbations, respectively. We also note a different behavior close to the unstable periodic orbit and far from it. For the small perturbation $\Delta\omega = 5^0$ there is a chaotic interchange between libration (with large amplitude) and rotation, while for the larger perturbation $\Delta\omega = 45^0$ we have a regular libration and for the still larger perturbation $\Delta\omega = 90^0$ we have a slow increase of $(\omega_2 - \omega_1)$, followed by a rotation for a short time, and this is repeated in a regular way. This behavior is clearly seen in the global view, by making use of LCN, given in Figure 1.16a. In this Figure we note that there are stable and unstable windows. The stable windows correspond to asymmetric configurations of the two planetary orbits. The stable motions in the Figures 1.14b,c are clearly in the stable windows of Figure 1.16a.

In Figure 1.16b we present a global view of the stable regions when the perturbation is ΔM. The behavior is similar to the $\Delta\omega$ perturbation. Close to the unstable orbit the motion appears as chaotic (but bounded), and far from the unstable periodic orbit ordered motion exists. Note that it this case the deviation from symmetry results to ordered motion, because we come close to families of stable asymmetric periodic orbits at the 3:1 resonance.

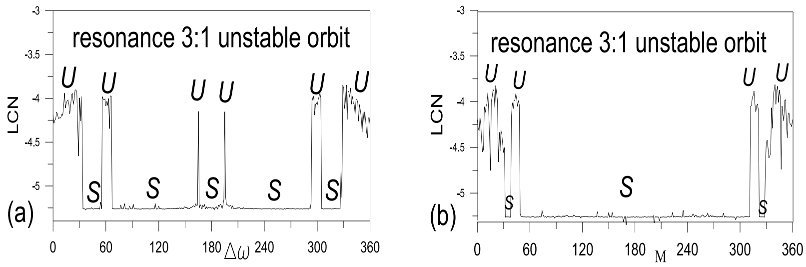

Fig. 1.16. A global view of the stable and unstable regions by the method of LCN: (a) Perturbation in $\Delta\omega$. (b) Perturbation in ΔM. Note the windows of stability, far from the unstable periodic orbit, corresponding to asymmetric motion.

1.6 A stable system at the 3:1 resonance. $(\omega_2 - \omega_1) = 0^0$.

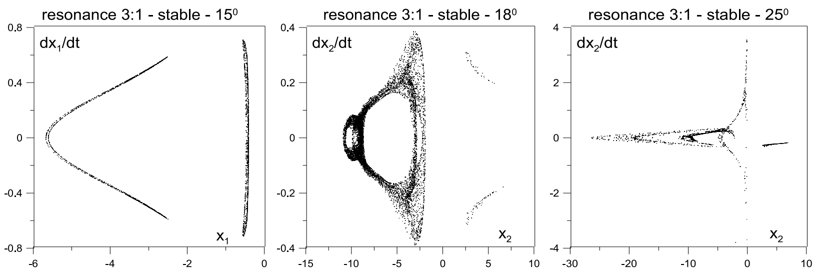

Fig. 1.17. The Poincaré map at the stable periodic orbit of Figure 1.13a. (a) Perturbation $\Delta\omega = 15^0$. The Poincaré map is bounded, on a torus. (b) Perturbation $\Delta\omega = 18^0$. The Poincaré map is still on a torus. (c) Perturbation $\Delta\omega = 25^0$. The Poincaré map is now chaotic and the planet P_2 escapes.

In this section we consider a 3:1 resonant *stable* periodic orbit, with large eccentricities, corresponding to a planetary system with masses $m_{SUN} = 0.99903$, $m_1 = 0.00078$, and $m_2 = 0.00019$, semimajor axes $a_1 = 3.041$AU, $a_2 = 6.453$AU ($T_2/T_1 = 3.091$) and eccentricities $e_1 = 0.853$, $e_2 = 0.664$. The orbit is symmetric, with the lines of apsides of the two planets aligned, $(\omega_2 - \omega_1) = 0^0$, and the two planets are at perihelion at $t = 0$, (see Figure 1.13b) and belongs to the stable part of a family of periodic orbits (Voyatzis and Hadjidemetriou 2006) for the masses of 55Cnc (the same family as in section 1.5). We work as in the previous case, and the results are presented in Figures 1.17 and 1.18. We start with a small perturbation $\Delta\omega = 15^0$ (Figure 1.17a) and increase the perturbation to $\Delta\omega = 18^0$ (Figure 1.17b) and to $\Delta\omega = 25^0$ (Figure 1.17c). Close to the stable periodic orbit ($\Delta\omega = 15^0$) the Poincaré map is on a well defined torus. The same is true for a further small increase of the perturbation $\Delta\omega = 18^0$, but for another small increase of the perturbation to $\Delta\omega = 25^0$ the motion is chaotic. Note that close to the stable periodic orbit we have ordered motion, but the region of stability is small, because in this case the planetary eccentricities are large. A similar behavior appears for the evolution of the angle $(\omega_2 - \omega_1)$, as presented in Figures 1.18a,b,c. For a small perturbation we start with a regular libration, but as the perturbation increases, an intermittent interchange between libration and rotation appears and for a further increase of the perturbation the evolution is chaotic. This behavior is clearly seen in the global view, by making use of LCN, given in Figure 1.19a. In this Figure we note that the only stable region is close to the stable periodic orbit, with a short range of stability. In Figure 1.19b we present a global view when the perturbation is in ΔM. The behavior is similar to the $\Delta\omega$ perturbation.

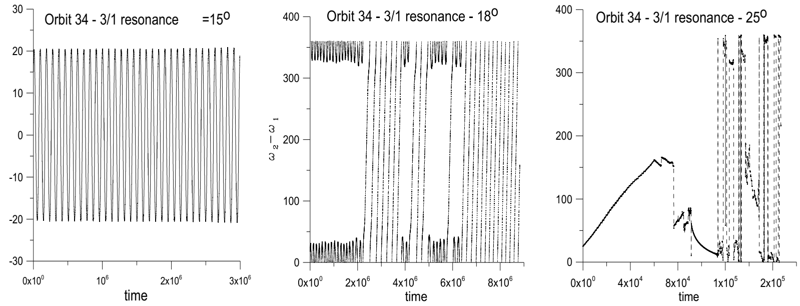

Fig. 1.18. The evolution of the angle $(\omega_2 - \omega_1)$ at the stable periodic orbit of Figure 1.13a. (a) Perturbation $\Delta\omega = 15^0$. There is a libration around 0^0. (b) Perturbation $\Delta\omega = 18^0$. There is a chaotic transition between libration and rotation, while the motion is bounded. (c) Perturbation $\Delta\omega = 25^0$. The evolution is chaotic.

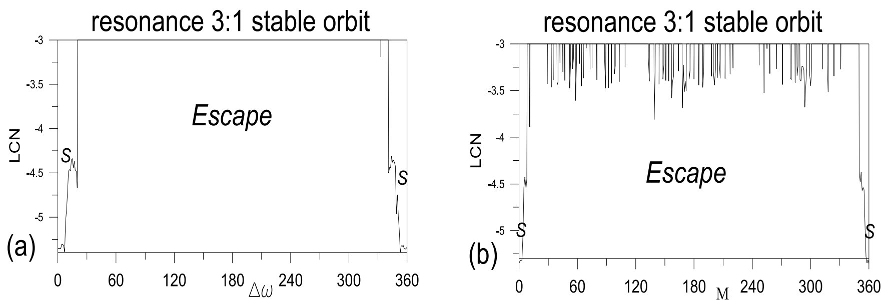

Fig. 1.19. A global view of the stable and unstable regions by the method of LCN: (a) Perturbation in $\Delta\omega$. (b) Perturbation in ΔM.

1.7 Discussion

The mean motion resonance of a planetary system with two planets depends only on the ratio of the semimajor axes. So, we may have an infinity of planetary systems, with *the same resonance*, but different eccentricities, different orientations of the planetary orbits, as defined by the angle $(\omega_2 - \omega_1)$ between the line of apsides (see Figure 1.1b) and different phases of the two planets, i.e. the position of the second planet on its orbit when the first planet is at perihelion, as defined by the the angle ΔM (see Figure 1.1c). In this work we studied the factors that affect the stability and the long term evolution of a *resonant* planetary system, keeping the resonance *fixed* and perturbing the system by changing the angles $\Delta\omega$ or ΔM. We presented four typical examples, starting from *symmetric periodic orbits* (sections 1.2-1.6) and we considered both stable and unstable periodic orbits as starting points. Special emphasis is given on the evolution of the angle $(\omega_2 - \omega_1)$ as the perturbation increases. The study was made by computing the Poincaré map and by computing the LCN for the whole range of perturbations. For a small perturbation close to a *stable* periodic orbit, we find a libration of all the orbital elements around their unperturbed values, and also the Poincaré map was on a well defined torus. The behavior however of a resonant system is not the same for larger deviations from the exact periodic motion, and this is more evident in the evolution of $(\omega_2 - \omega_1)$. In some cases (section 1.3) we have a smooth transition from libration to rotation, before the system develops chaotic motion for still larger perturbations. In other cases (section 1.4) we start with a libration of $(\omega_2 - \omega_1)$, but then we go directly to chaotic motion, as the perturbation increases, without passing through rotation. In a third typical case (section 1.6), we start with libration of $(\omega_2 - \omega_1)$ but then, as the perturbation increases, an intermittent transition between libration and rotation appears, while the system remains bounded. For still larger perturbations we have chaotic motion, as in all previous cases. In all the above cases where we had bounded motion, the Poincaré map is on a well defined torus. The situation is different in the vicinity of an *unstable* periodic motion. For a small perturbation, the motion close to the unstable periodic orbit is bounded (section 1.5) but the Poincaré map is no longer on a torus, but its boundary seems to be a *fractal*. The evolution of $(\omega_2 - \omega_1)$ is also chaotic, presenting an intermittent interchange between libration and rotation. For a larger perturbation, further from the unstable periodic orbit, ordered motion appears and the Poincaré map is now on a well defined torus. The evolution of $(\omega_2 - \omega_1)$ in this latter case is represented by regular motion, either libration or rotation (according to the perturbation). All these results, which were presented by the Poincaré map and the evolution of the elements of the orbit, were verified by a systematic study of the stability of an orbit by the method of LCN. In this way we also detected regions in the phase space where we have stable resonant motion, but *non symmetric*.

From all the above we see that the resonance alone is not enough to determine the stability at a *fixed* resonance, but there are other factors that determine the stability, as the deviation from symmetry, defined by $\Delta\omega$ and the change of the phase between the planets, defined by ΔM. We also noted that the chaotic property is not necessarily associated with the disruption of the system, and the system may be bounded, even if the LCN (or similar chaotic indicators) show chaos. In this latter case the chaotic property may appear in a different way: Some elements of the orbit, notably the angle $(\omega_2 - \omega_1)$, may have a chaotic evolution and the system may move in a region of the phase space with fractal dimensions.

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