The 1/1 Resonance in Extrasolar Planetary Systems

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Abstract

We study orbits of planetary systems with two planets, for planar motion, at the 1/1 resonance. This means that the semimajor axes of the two planets are almost equal, but the eccentricities and the position of each planet on its orbit, at a certain epoch, take different values. We consider the general case of different planetary masses and, as a special case, we consider equal planetary masses. We start with the *exact resonance*, which we define as the 1/1 resonant *periodic* motion, in a rotating frame, and study the topology of the phase space and the long term evolution of the system in the vicinity of the exact resonance, by rotating the orbit of the outer planet, which implies that the resonance and the eccentricities are not affected, but the symmetry is destroyed. There exist, for each mass ratio of the planets, two families of symmetric periodic orbits, which differ in phase only. One is stable and the other is unstable. In the stable family the planetary orbits are in antialignment and in the unstable family the planetary orbits are in alignment. Along the stable resonant family there is a smooth transition from planetary orbits of the two planets, revolving around the sun in eccentric orbits, to a close binary of the two planets, whose center of mass revolves around the sun. Along the unstable family we start with a collinear Euler - Moulton central configuration solution and end to a planetary system where one planet has a circular orbit and the other a Keplerian rectilinear orbit, with unit eccentricity. It is conjectured that due to a migration process it could be possible to start with a 1/1 resonant periodic orbit of the planetary type and end up to a satellite-type orbit, or vice versa, moving along the stable family of periodic orbits.

keywords: periodic orbits, 1/1 resonance, stability.

1 Introduction

The 1/1 resonance in a planetary system is a particular case of a resonant extrasolar planetary system. By the term 1/1 resonance we usually mean the motion of Trojan-like asteroids, which are fixed points in a rotating frame, where the planet is on the rotating x-axis, and consequently both the planet and a Trojan asteroid revolve around the Sun with the same period (Goździewski and Konacki, 2006; Dvorak et al., 2007; Dvorak et al., 2008). In this study we consider a different type of 1/1 mean motion resonance in a planetary system.

Let us consider a planetary system with two planets, P_1, P_2, and start with the case where the two planetary masses are equal to zero, $m_1 = m_2 = 0$. We

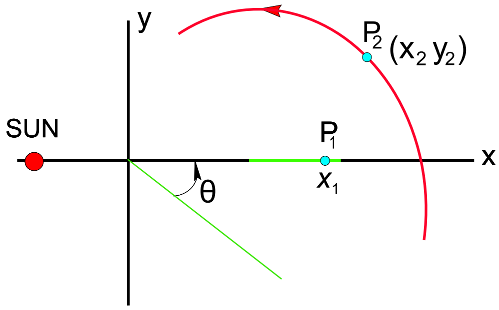

Figure 1: The rotating frame xOy. The planet P_1 moves on the x-axis and the planet P_2 moves in the xOy plane. The angle θ defines the orientation of the rotating frame and is ignorable.

restrict the study to planar motion and assume that the semimajor axes of the planets are equal, $a_1 = a_2$. In this case the motion is periodic in an inertial frame, for any value of the eccentricities and the angles ω_1, ω_2 of the apsidal lines and also for arbitrary position of the two planets on their orbits at $t = 0$. The question is what happens when the masses of the planets are no longer equal to zero?

When we give masses to the two planets, we have the model of the *general three body problem* and it is convenient to study the motion in a *rotating* frame, whose x-axis is the line Sun-P_1, its origin is at the center of mass of these two bodies and the planet P_2 moves in the xOy plane (Fig. 1). We have four degrees of freedom, and we take as coordinates the position x_1 of P_1 on the x-axis, the coordinates x_2, y_2 of P_2 in the xOy plane and the angle θ between the x-axis and a fixed direction in the inertial frame. It turns out (Hadjidemetriou 1975) that the angle θ is ignorable, so we can restrict the study to the rotating frame only. This means that the degrees of freedom are reduced to three and the phase space is six dimensional. It is also known that families of periodic orbits of the general three body problem exist in the *rotating* frame xOy, which means that the *relative* configuration is repeated in the inertial frame. In particular, families of resonant periodic orbits of the planetary problem, for several mean motion resonances, exist (see e.g. Beaugé et al., 2003; Hadjidemetriou, 2002, 2006; Voyatzis and Hadjidemetriou, 2005,2006; Michtchenko et al., 2006; Psychoyos and Hadjidemetriou, 2005; Ji et al., 2005, 2007; Voyatzis, 2008).

It is evident that the above mentioned 1/1 resonant unperturbed periodic orbits in the inertial frame, are also periodic in the rotating frame. In this frame the planet P_1 moves on the x-axis and the planet P_2 describes periodic motion in the xOy plane. Since, as mentioned above, the periodic orbits exist in the *rotating* frame, we refer the above mentioned unperturbed 1/1 resonant periodic motion to the rotating frame xOy. Let us now give masses to the two planets. It turns out that, as we will see in the following, out of the infinite set of resonant periodic orbits that we had for $m_1 = m_2 = 0$, only two orbits survive

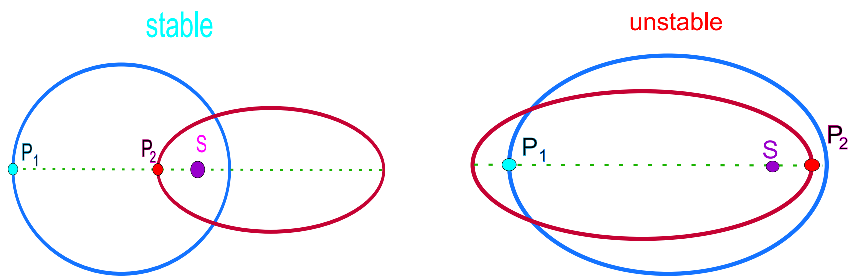

Figure 2: (a) The stable configuration of the 1/1 resonant periodic orbits. The planetary orbits are in antialignment and the two planets P_1, P_2 are in aphelion and perihelion, respectively, at $t = 0$. (b) The unstable configuration at the 1/1 resonance. The planetary orbits are in alignment and the two planets P_1, P_2 are also in aphelion and perihelion, respectively, at $t = 0$.

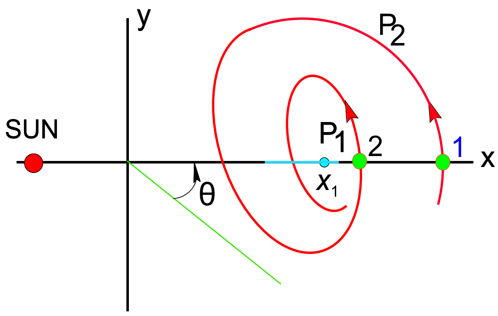

Figure 3: The Poincaré map on the surface of section $y_2 = 0$, $(\dot{y}_2 > 0)$, $H = h =$ constant. The consecutive points of intersection are 1,2,...

the perturbation, for a given energy, which are both symmetric with respect to the x-axis. One of them is stable and the other is unstable, as we shall see in the following. In the stable case the planetary orbits are in antialignment $(\omega_2 - \omega_1 = 180°)$ and at $t = 0$ P_1 is at aphelion and P_2 at perihelion. In the unstable case the planetary orbits are in alignment $(\omega_2 - \omega_1 = 0°)$, while at $t = 0$ P_1 is at aphelion and P_2 at perihelion, as in the stable case (Fig. 2). Note that due to the 1/1 resonance, after half a period the position of perihelion and aphelion are interchanged, which implies that the configurations $\{P_1$: aphelion - P_2: perihelion$\}$ and $\{P_1$: perihelion - P_2: aphelion$\}$ are equivalent.

The topology of the phase space and the properties of motion close to a periodic orbit are studied by computing the Poincaré map on a surface of section (Hadjidemetriou, 2006; Voyatzis, 2008). By the Poincaré map we reduce the dimensions of the phase space, without losing the generality of the problem, and in addition we eliminate the unnecessary details that are not important in the study of the long term evolution of the system. In the present study, in all our computations, we consider the surface of section

$$y_2 = 0 \ (\dot{y}_2 > 0), \ \ H = h = \text{constant}.$$

This means that we consider the consecutive intersections of the planet P_2 with the x-axis, in the same direction, (Fig. 3) and consequently the reduced phase space of the Poincaré map is the four dimensional space $x_1, \dot{x}_1, x_2, \dot{x}_2$. In presenting the diagrams of the Poincaré map, we give the projection on a coordinate plane.

In the following sections we compute families of periodic orbits, at the 1/1 resonance, for the general case $m_1 \neq m_2$ and also for the special case $m_1 = m_2$. Note that, contrary to other mean motion resonances, for example 2/1, 3/1, 5/2, ..., in the present case of the 1/1 resonance the distinction between inner planet and outer planet does not have any meaning. So, we call arbitrarily the one planet, planet P_1 and the other planet, planet P_2. It is also clear that the case $m_1 > m_2$ includes also the case $m_1 < m_2$ (by reversing the names of P_1 and P_2), and for this reason we studied only the case $m_1 > m_2$. As a special case, we also considered $m_1 = m_2$.

In all our computations we defined the units of mass, length and time by the normalizing conditions

$$m_0 + m_1 + m_2 = 1, \quad G = 1, \quad L = \text{constant},$$

where m_0 is the mass of the sun, G is the gravitational constant and L is the angular momentum constant. This latter condition comes in a natural way in our computations, because when we use the ignorable angle θ to reduce the degrees of freedom from four to three and restrict the motion to the rotating frame xOy only (Fig. 1), the angular momentum L appears as a fixed parameter in the reduced equations of motion in the rotating frame (Hadjidemetriou, 1975).

2 Families of 1/1 resonant periodic orbits for $m_1 \neq m_2$

From the infinite set of 1/1 resonant periodic orbits, for $m_1 = m_2 = 0$, both symmetric and asymmetric, that we mentioned in the Introduction, only two *symmetric* monoparametric families survived when $m_1 > 0$, $m_2 > 0$. The non zero initial conditions of a symmetric periodic orbit, in the rotating frame xOy, are $x_{10}, x_{20}, \dot{y}_{20}$, because it is $\dot{x}_1 = 0$, $y_{20} = 0$ and $\dot{x}_{20} = 0$, due to the symmetric property (the planet P_2 starts perpendicularly from the rotating x-axis and at the same time the planet P_1 is temporarily at rest on the x-axis). Consequently, a family of symmetric periodic orbits is represented by a smooth curve in the space of (non zero) initial conditions $x_{10}, x_{20}, \dot{y}_{20}$. However, in order to have a better physical understanding of these families, we present them in the space of the initial osculating eccentricities of the two planets at $t = 0$ (although this

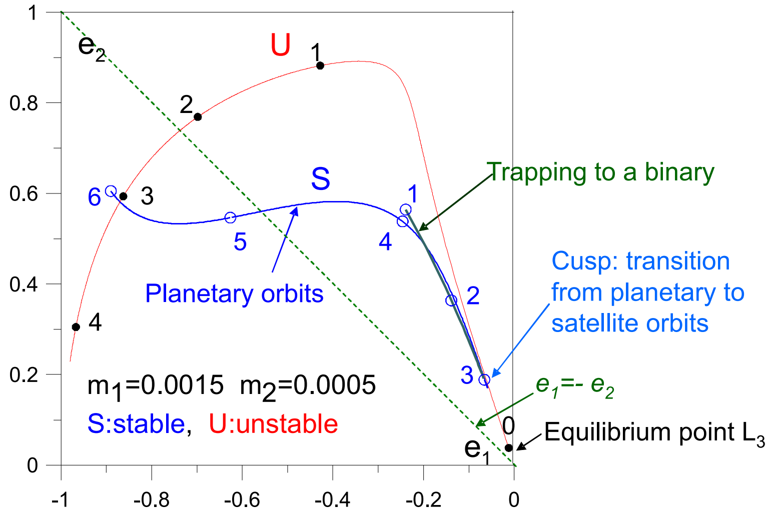

Figure 4: The families of stable and unstable symmetric periodic orbits, in the $e_1 - e_2$ space. The stable family is indicated by S and the unstable by U. At $t = 0$ the planet P_1 is at perihelion ($e_1 < 0$) and the planet P_2 is at aphelion ($e_2 > 0$). The cusp in the stable family, at the orbit 3, marks the division of the family into two parts: satellite orbits (orbits 1, 2, 3) and planetary orbits (orbits 4, 5, 6). The line $e_1 = -e_2$ is drawn and its intersections with the two families define the periodic orbits with equal eccentricities.

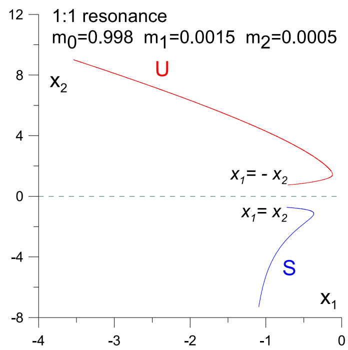

Figure 5: The families of stable and unstable symmetric periodic orbits, in the x_1x_2 space. The one end of the unstable family, at $x_1 = -x_2$ corresponds to the equilibrium orbit 0 in Fig. 4. The one end of the stable family, at $x_1 = x_2$, corresponds to the close binary ORBIT 1 in Fig. 4.

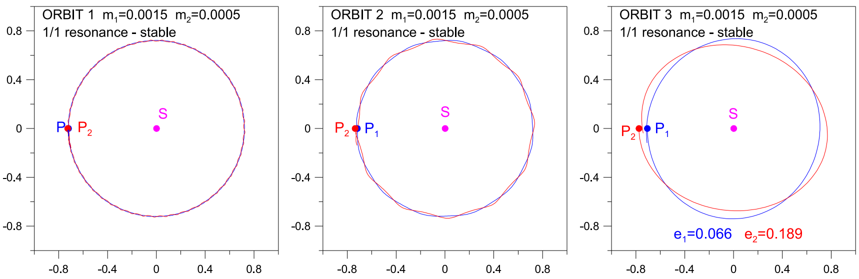

Figure 6: The satellite orbits corresponding to the orbits 1, 2, 3 on the stable family S of Fig. 4. The orbit 3 is the transition orbit between satellite and planetary orbits.

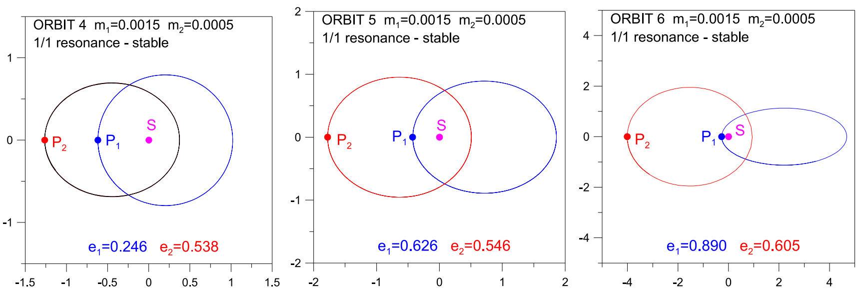

Figure 7: The planetary orbits corresponding to the orbits 4, 5, 6 on the stable family S of Fig. 4.

presentation does define accurately a periodic orbit). We used the convention that $e_i > 0$ means position at aphelion and $e_i < 0$ position at perihelion.

In Fig. 4 we present, in the $e_1 - e_2$ space, the two symmetric families of periodic orbits, for the masses

$$m_0 = 0.998, \quad m_1 = 0.0015, \quad m_2 = 0.0005.$$

In Fig. 5 we give the projection of the families on the $x_{10} - x_{20}$ plane. One family is stable and the other is unstable and there is no change of the linear stability along the families.

Let us focus our attention to the presentation of the families in Fig. 4. The orbits of the stable family, indicated by S, are in antialignment ($\omega_2 - \omega_1 = 180°$) and at $t = 0$ the planet P_1 is at perihelion and P_2 at aphelion. Along the stable family there is a cusp at the orbit 3, which divides the family into two parts. The part which contains the orbits 4, 5, 6 corresponds to the case where the gravitational attraction from the sun dominates and the gravitational interaction between the two planets is a perturbation. In this case the orbits are of the *planetary type* (Fig. 7). On the contrary, the part which contains the orbits 1, 2, 3 corresponds to the case where the gravitational interaction between the two planets dominates, and consequently the two planets are trapped into a close binary whose center of mass revolves around the sun (Fig. 6). We call these orbits *satellite type* orbits.

We remark that in the presentation of the periodic orbits in figure 4 we use the *oscullating* elements. These are computed by considering the position and velocity of each planet in the inertial frame, at $t = 0$, assuming that these are the initial conditions that determine the Keplerian orbit of the planet around the sun. This is true if any other factors that affect the velocity (in our case the gravitational interaction between the planets) are negligible, compared to the attraction from the sun. However, this is not true in the part of the family corresponding to satellite orbits, because the velocity of each planet is greatly affected by the gravitational interaction of the two planets, that are very close to each other. Consequently, the computation of the Keplerian oscullating eccentricities is not correct. In this sense, the presentation of the satellite part of the stable family is meaningless in the space of the osculating eccentricities, and we draw it just to show the transition from planetary to satellite orbits. The transition point is the cusp in Fig. 4. This cusp does not exist in the presentation of the family in the mathematical space of initial conditions, as shown in Fig. 5., indicating that S is a continuous family of periodic orbits.

Note also from Fig. 4, that on the stable family there exists a periodic orbit where the eccentricities of the two planets are equal. In general, the stable family starts from a close binary which revolves around the sun and evolves to planetary orbits with large eccentricities.

We remark that the orbits in the figures 6 and 7 are presented in the inertial frame. However, these orbits are exactly periodic in the *rotating frame* only (Fig. 1) and in the inertial frame they appear as ellipses that rotate. For this reason, the orbits in figures 6, 7 are drawn for one period only, where the rotation in space is not apparent. The same is true for all other orbits that we present bellow.

We consider now the unstable family, as presented in Fig. 4, in the space $e_1 - e_2$. The planetary orbits are in alignment, while P_1, P_2 are in perihelion and aphelion, respectively (and vice versa after half a period). Contrary to the stable family, in this case the two planets never come close to each other and, consequently, the gravitational attraction from the sun dominates all along the family. The family is represented by a smooth curve and all members of the family are of the planetary type. There is no trapping into a close binary, as in the stable case.

In figures 8 and 9 we present some typical orbits along the unstable family. In Fig. 8 we present the orbits 0, 1 and 2, as shown in Fig. 4. Note that the orbit 0 in fact represents an equilibrium configuration, where the two planets move on almost the same circular orbits around the sun, at opposite phases. Note also that along the unstable family there exists an orbit where $e_1 = -e_2$, as seen in Fig. 4 (intersection of the family with the line $e_1 = -e_2$). An orbit very close to this is the orbit 2 in Fig. 8, where the two planets move on almost the same elliptic orbit, at opposite phases (perihelion - aphelion, respectively

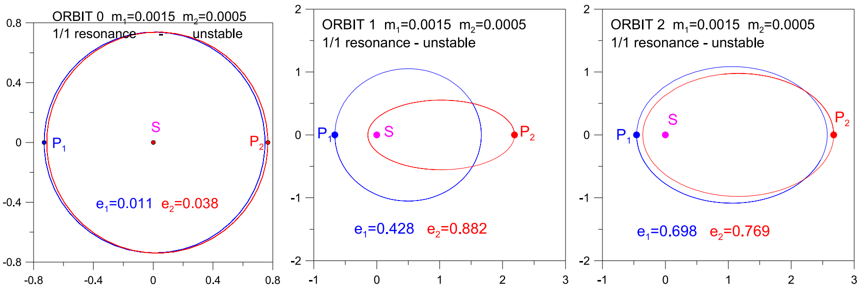

Figure 8: The planetary orbits corresponding to the orbits 0, 1, 2 on the unstable family U of Fig. 4. The orbit 0, at the one end of the family corresponds to an equilibrium configuration. In orbit 2 the eccentricities are close to each other. In this case the two planets move almost on the same orbit, in opposite phases, as in the equilibrium orbit 0.

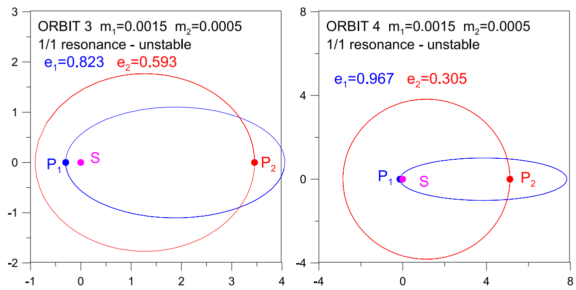

Figure 9: The planetary orbits corresponding to the orbits 3, 4 on the unstable family U of Fig. 4.

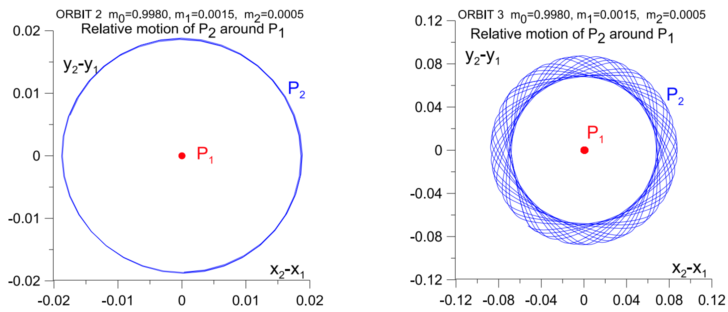

Figure 10: (a) The relative motion of P_2 around P_1 for the satellite orbit 2 of the stable family (Fig.6b). The orbit is exactly circular. (b) The relative motion of P_2 around P_1 for the orbit 3 of the stable family, which is close to the transition from satellite to planetary orbits (Fig. 6c).

at $t = 0$). In Fig. 9 we present two more unstable orbits, orbit 3 and orbit 4. Comparing the unstable orbits of figures 8 and 9 with the unstable family in Fig. 4, we note that the family starts (orbit 0) with a collinear Euler - Moulton central configuration solution (Meyer, 1999) and ends with an orbit where the eccentricity of the planet P_1 goes to a rectilinear orbit with eccentricity equal to one, while the eccentricity of the planet P_2 goes to a circular orbit.

In order to have a better geometrical view of the satellite orbits of the stable family (Fig.4) that are shown in figure 6, we plot the relative motion of one planet around the other. In Fig.10a we present the relative motion of the planet P_2 around the planet P_1 for the satellite orbit 2. The orbit is exactly circular. The same is true for all other satellite orbits. In Fig.10b we present the relative motion of P_2 around P_1 for the orbit 3. This is close to the transition orbit between satellite and planetary orbits.

3 The evolution in the vicinity of a resonant periodic orbit

In this section we study the long term evolution close to a 1/1 resonant periodic orbit. For this reason we consider a typical periodic orbit, orbit 4, on the stable family of Fig. 4. We start with the exact periodic orbit and we change the initial conditions by rotating the orbit of the planet P_2 by an angle $\Delta\omega$ and keeping the eccentricities and the semimajor axes the same as in the exact resonance. The new orbit is also at the 1/1 resonance, but is no longer periodic. This rotation is made at $t = 0$, when the planet P_1 is at perihelion and the planet P_2 at aphelion. In order to study, in addition to the above change of the initial conditions, the effect of the mass ratio of the planetary orbits, we consider two cases: In the first case we also change the mass of the outer planet P_2 from $m_2 = 0.0005$ to

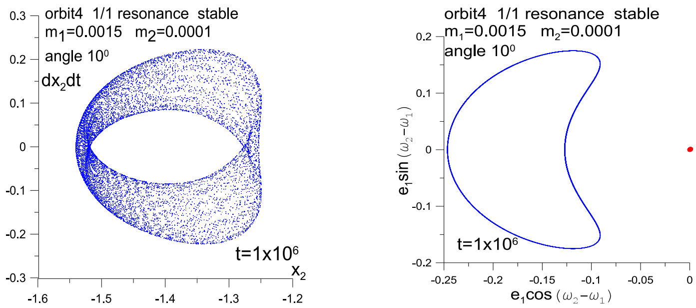

Figure 11: The case of to $m_2 = 0.0001$ and the change of the initial conditions of the exact periodic orbit 4, by rotating the orbit of the planet P_2 by an angle $\Delta\omega = 10°$: (a) The Poincaré map, projection on the $x_2 - \dot{x}_2$ plane. The motion is on a torus. (b) The evolution of the angle $\omega_2 - \omega_1$, in the plane $e_1 \cos(\omega_2 - \omega_1)$-$e_1 \sin(\omega_2 - \omega_1)$. The angle $\omega_2 - \omega_1$ librates. The angle of libration is small, since the center of libration, defined as the point $(0, 0)$ (indicated by the dot) is far.

$m_2 = 0.0001$ (the total mass is equal to unity) and we checked that the elements of the two planetary orbits are not affected essentially from this mass change. In the second case we keep the mass of P_2 equal to $m_2 = 0.0005$, as in the exact periodic orbit.

3.1 The stable family

The case $m_2 = 0.0001$

As a typical example we consider the orbit 4 of the stable family (Fig. 4 and Fig. 7a), with eccentricities $e_1 = 0.246$ and $e_2 = 0.538$. We start with an angle of rotation $\Delta\omega = 10°$ and increase this angle to $\Delta\omega = 30°$, $\Delta\omega = 40°$, $\Delta\omega = 42°$, $\Delta\omega = 50°$ and $\Delta\omega = 55°$. The results are shown in figures 11-16. In each figure we give the Poincaré map (projection on the coordinate plane $x_2 - \dot{x}_2$) and the evolution of the angle between the line of apsides, $\omega_2 - \omega_1$, of the two planetary orbits. We note that up to a deviation of the initial conditions equal to $\Delta\omega < 40°$, the motion is on a well defined torus, and the angle $\omega_2 - \omega_1$ *librates* with an amplitude which increases as $\Delta\omega$ increases from $0°$ to $40°$. Just beyond the value $\Delta\omega = 40°$, the motion still remains on a torus, but now the angle $\omega_2 - \omega_1$ *rotates*. The transition is clearly seen by comparing the figures 13b and 14b. The motion remains bounded up to $\Delta\omega = 50°$, and for larger values the system is captured temporarily on a torus, before chaotic motion appears and the system disrupts (Fig. 16). In Figures 15a and 16a new intersection points appear on the Poincaré map, compared to the Figures 11a - 14a. This does not mean that the dynamics has changed, but is due to a geometrical property: the surface of section is the rotating x-axis and for $\Delta\omega > 50°$ (and after some time) the perturbed orbit presents new intersections with this axis.

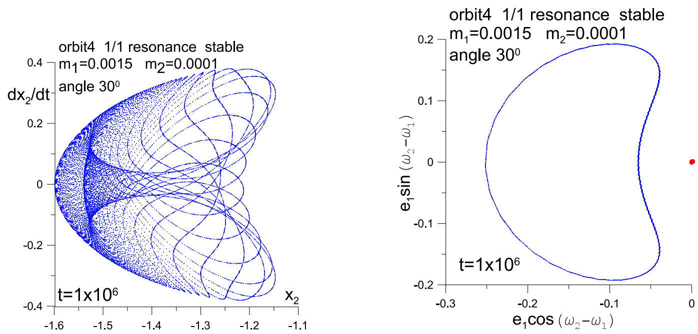

Figure 12: As in Fig. 11 for $\Delta\omega = 30°$. The motion is on a torus but it seems to be close to a multiple periodic orbit. The angle $\omega_2 - \omega_1$ librates with an angle larger than in the case $\Delta\omega = 10°$, because the center of libration $(0,0)$ is closer.

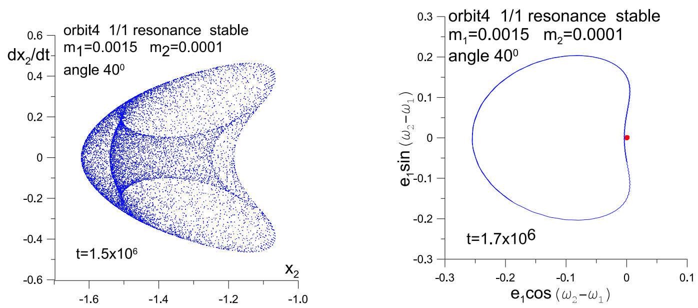

Figure 13: As in Fig. 11 for $\Delta\omega = 40°$. The motion is on a torus. The angle $\omega_2 - \omega_1$ librates with a large angle, because the center of libration $(0,0)$ is very close to the libration invariant curve.

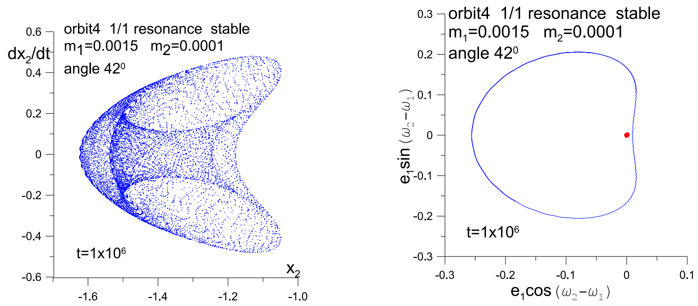

Figure 14: As in Fig. 11 for $\Delta\omega = 42°$. The motion is still on a well defined torus. The angle $\omega_2 - \omega_1$ rotates. The transition from libration to rotation is clearly seen, by comparing this figure with figure 13b (the point $(0, 0)$ is now inside the invariant curve).

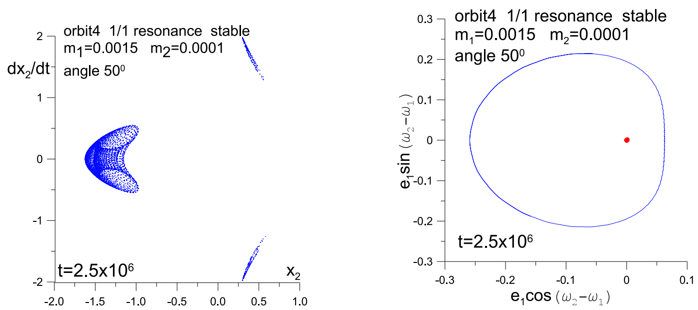

Figure 15: As in Fig. 11 for $\Delta\omega = 50°$. The motion is still on a torus, whose topology has now changed, compared to the torus on Fig. 14. The angle $\omega_2 - \omega_1$ rotates.

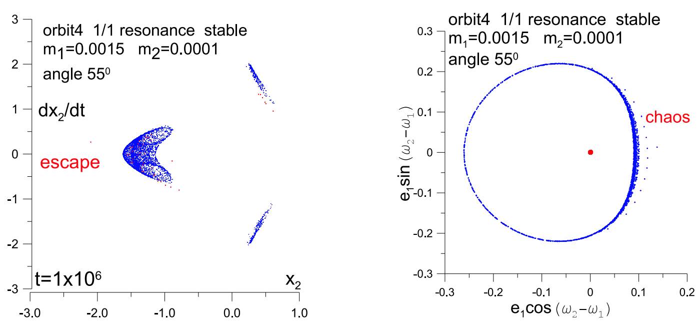

Figure 16: As in Fig. 11 for $\Delta\omega = 55°$. The motion is temporary captured on a torus, but later chaotic motion appears. The angle $\omega_2 - \omega_1$ rotates at the beginning, but later chaotic motion appears.

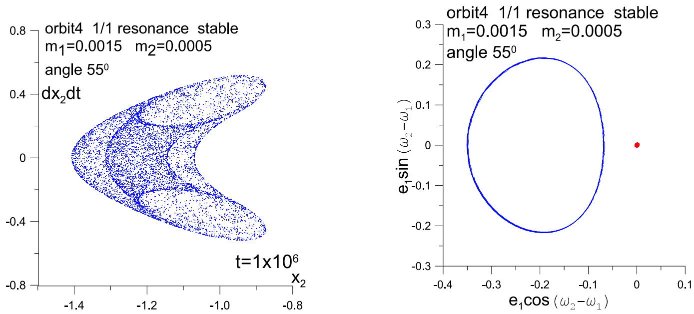

Figure 17: The case $m_2 = 0.0005$ and $\Delta\omega = 55°$ The motion is ordered and the angle $\omega_2 - \omega_1$ librates.

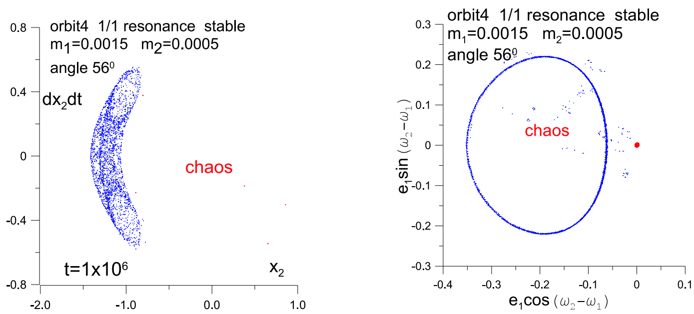

Figure 18: As in Fig. 17 for $\Delta\omega = 56°$. The motion is first trapped on a torus but later chaotic motion appears. The angle $\omega_2 - \omega_1$ at first librates, but later chaotic motion appears.

Concerning the behavior of the angle $\omega_2 - \omega_1$ between the line of apsides of the two planets, we note that there is a smooth transition from libration to rotation, as the change $\Delta\omega$ of the initial conditions increases, while the motion remains bounded. We remind that the rotation $\Delta\omega$ is at $t = 0$, when P_1 is at perihelion and P_2 at aphelion. This implies that libration and rotation of a resonant, but non periodic, planetary orbit are not different types of motion, but are closely related.

The case $m_2 = 0.0005$

We repeat now the same work as before, but now the mass of m_2 is equal to $m_2 = 0.0005$, as in the computed family of periodic orbits in Figure 4. Up to $\Delta\omega = 55°$ the motion is on a torus and the angle $\omega_2 - \omega_1$ librates (Figure 17). As soon as the rotation angle increases to $\Delta\omega = 56°$, chaotic motion appears after a long time where the motion is trapped on a torus. The angle $\omega_2 - \omega_1$ at first librates, but later chaotic motion appears (Figure 18). Note that in this

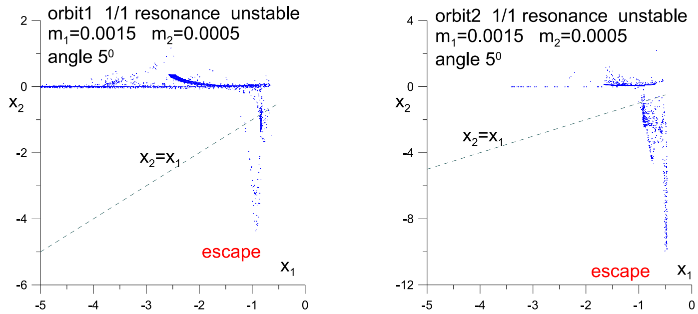

Figure 19: The projection of the Poincaré map on the $x_1 - x_2$ plane, for a change of the initial conditions of the exact unstable periodic orbit, by rotating the orbit of the planet P_2 by an angle $\Delta\omega = 5°$. In both cases strongly chaotic motion appears very soon, due to close encounters between the two planets, as it is seen from the points of the Poincaré map close to the line $x_2 = x_1$: (a) The evolution of the orbit 1 (Fig. 8b). (b) The evolution of the orbit 2 (Fig. 8c).

case the angle $\omega_2 - \omega_1$ goes directly from libration to chaos, without passing through rotation, as it happened in the previous case $m_2 = 0.0001$.

3.2 The unstable family

All the periodic orbits of the unstable family (Fig. 4) are highly unstable and a small deviation from the exact periodic orbit results to chaotic motion in a short time. In figures 19a,b we present the evolution of two typical periodic orbits on the unstable family, orbits 1 and 2, shown in figures 8b and 8c, respectively. In both cases the change of the initial conditions was $\Delta\omega = 5°$. The Poincaré map is given by the projection on the $x_1 - x_2$ plane, because it is in this projection that the mechanism of generation of chaos is clearly seen. As we see, some points of intersection are very close to the line $x_2 = x_1$, which implies a close approach between the two planets (it is a real encounter and not just a result of the projection, because P_1 is always on the x-axis and $y_2 = 0$ for all the points of the Poincaré map).

4 The special case $m_1 = m_2$

In this section we present the families of stable and unstable symmetric periodic orbits for the special case $m_1 = m_2$. This introduces some additional symmetries to the model, compared to the general case $m_1 \neq m_2$. The two families of stable and unstable symmetric periodic orbits, in the $e_1 - e_2$ space of initial osculating eccentricities, are presented in Fig. 20. As in the general case, the planetary

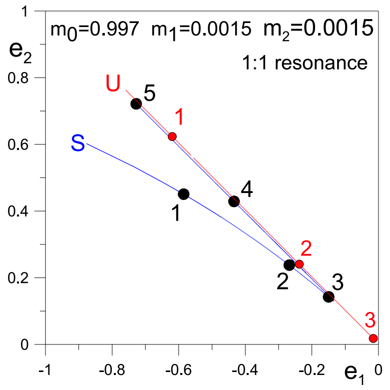

Figure 20: The families of stable and unstable symmetric periodic orbits, in the $e_1 - e_2$ space. The stable family is indicated by S and the unstable by U. Both families (with the exception of the part of the stable family before the cusp), lie on the line $e_1 = e_2$, due the special symmetry $m_1 = m_2$.

orbits on the stable family are in antialignment and at $t = 0$ P_1 is at perihelion and P_2 at aphelion (and vice versa at $t = T/2$, after half a period).

In the present case, due the special symmetry $m_1 = m_2$, both families, as presented in the space $e_1 - e_2$, lie on the line $e_1 = -e_2$, with the exception of the part of the stable family before the cusp (orbits 1, 2, 3). This means that *all* orbits, both stable and unstable, have the same eccentricities, contrary to the general case where only one orbit of each family has equal eccentricities (the ones that corresponds to the intersection of the family with the line $e_1 = -e_2$ in Fig. 4). The cusp mentioned above is explained in the following section.

4.1 The stable family for $m_1 = m_2$

All orbits of the stable family are symmetric and in antialignment. The stable family is divided into two parts, by a cusp that corresponds to the orbit 3 in Fig. 20 (see also Fig. 21c). This is explained as follows: When the eccentricities of the planets are small, the gravitational interaction between the planets dominates the attraction from the sun. This is due to the fact that when P_1 is at perihelion, P_2 is at aphelion and the orbits are almost circular, the two planets are very close to each other and are trapped into a *close binary* whose center of mass revolves around the sun. These periodic orbits are of the *satellite type* and some typical orbits of this type are shown in Fig. 21. We remark that the osculating elements in the presentation of this part of the family in Fig. 20, the one containing the orbits 1, 2, 3, are *meaningless*, and we present this part of the family in Fig. 20 just to show the transition from satellite to planetary orbits. In the other part of the stable family, containing the orbits 4, 5, the planetary eccentricities are larger and the gravitational attraction from the sun

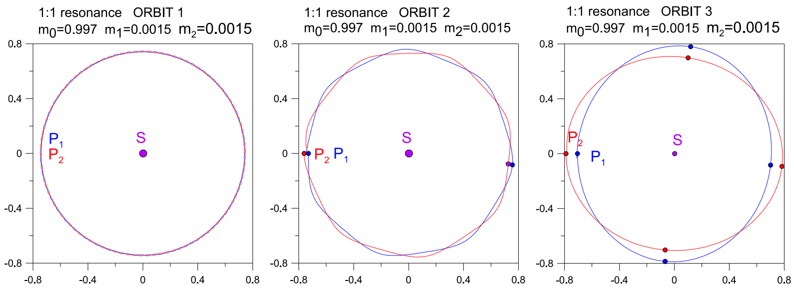

Figure 21: Three periodic orbits of the stable family, corresponding the orbits of the satellite type. The numbers of the orbits correspond to the points indicated in Fig. 20.

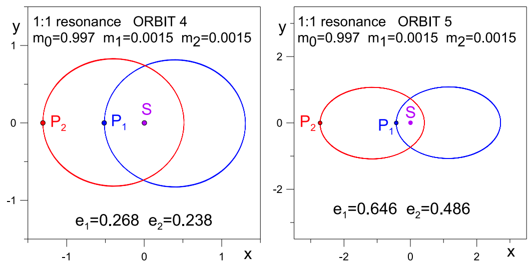

Figure 22: Two periodic orbits of the stable family, corresponding the orbits of the planetary type. The numbers of the orbits correspond to the points indicated in Fig. 20.

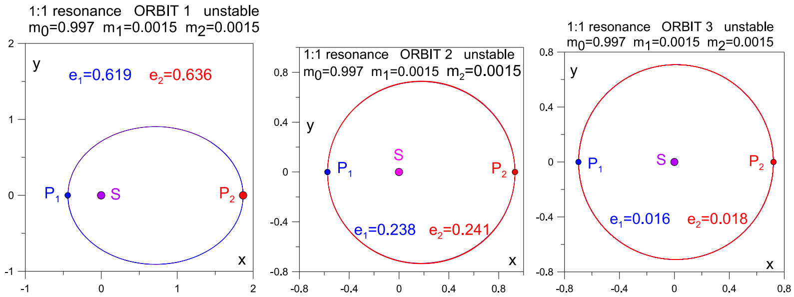

Figure 23: Three periodic orbits of the unstable family. All are of the planetary type. The numbers of the orbits correspond to the points indicated in Fig. 20.

dominates the gravitational interaction between the two planets. These orbits are of the *planetary type*. In Fig. 22 we present two typical periodic orbits with larger eccentricities. Note that in all planetary type orbits the eccentricities of the planets are almost equal, but the individual orbits do not coincide, because the planets are in antialignment. The orbit 3 in Fig. 21 is the transition orbit between satellite and planetary orbits.

The family of stable periodic orbits in this special symmetry, $m_1 = m_2$, starts from a close binary whose center of mass revolves around the sun (Fig. 21a) and ends to a planetary system with equal eccentricities (Fig. 22b), and it seems that at the end the planetary orbits are rectilinear orbits with eccentricities equal to 1.

All orbits of the stable family have around them a region of bounded motion, as in the general case studied in section 3. The results are similar to the general case and we do not repeat them here.

4.2 The unstable family for $m_1 = m_2$

The family of unstable periodic orbits is presented in Fig. 20, in the space of the eccentricities $e_1 - e_2$. This family lies on the line $e_1 = -e_2$, and all orbits are in alignment. This means that the two planetary orbits almost coincide, implying that both planets move on the *same* orbit, at opposite phases, perihelion - aphelion (at $t = 0$). Due to this configuration the two planets never come close to each other. Three typical periodic unstable orbits are shown in Fig. 23.

The unstable family starts from the collinear Euler - Moulton central configuration solution as shown in Fig. 23a and ends to a planetary system where the two planets are on the same orbit at opposite phase, with eccentricities ending to unity (straight line Keplerian orbits).

All the orbits of the unstable family are *highly* unstable and a very small deviation from the exact initial conditions results to strongly chaotic motion in a short time. The behavior is the same as in the general case $m_1 \neq m_2$ shown in section 4.

5 Discussion

There exist two families of resonant symmetric periodic orbits at the 1/1 resonance, both in the general case $m_1 \neq m_2$ and the special case $m_1 = m_2$. The periodicity refers to a *rotating* frame (Fig. 1), which means that the *relative* configuration is repeated in the inertial frame.

The planetary orbits in the inertial frame are almost ellipses with eccentricities that vary along each family. All types of orbits appear, including circular orbits and rectilinear orbits . Since all orbits are symmetric, the only possibility is for the two planetary orbits to be in alignment or antialignment. One family is stable and the other is unstable. The stability along the family does not change. This means that there are no critical points on the family (points where the stability changes) and consequently there is no any bifurcation to new families of 1/1 resonant periodic orbits. The stability depends on the relative position of the two planetary orbits.

In the stable family all orbits are in antialignment and in addition, at $t = 0$, when one planet, P_1 is at perihelion, the other planet, P_2, is at aphelion. Due to the 1/1 resonance, this initial configuration is equivalent to the configuration P_1 at aphelion and P_2 at perihelion.

The stable family, both in $m_1 \neq m_2$ and $m_1 = m_2$, is separated into two parts, in the space of initial osculating eccentricities $e_1 - e_2$ (see figures 4 and 20). One part represents *satellite* orbits (a close binary whose center of mass revolves around the sun) and the other part represents *planetary* orbits (two planets revolving around the sun in perturbed Keplerian orbits), as explained in detail in the previous sections. Around the stable planetary orbits at the 1/1 resonance there exists a region in the phase space where we have bounded motion on a torus. It is also worth noting that if we deviate from the initial conditions of the exact resonance in such a way that the 1/1 resonance is preserved, we have bounded resonant motion where the angle of apsides between the planetary orbits librates or rotates. This means that libration and rotation of a non periodic resonant planetary system is not a different type of motion, but are related by a continuous change of initial conditions.

In the unstable family the planetary orbits are in alignment and at $t = 0$ one planet is at perihelion and the other is at aphelion at $t = 0$ (and vice versa after half a period). All orbits on the unstable family are of the planetary type, since due to the present configuration the two planets cannot come close to each other. All these orbits are highly unstable, and a small deviation from the exact periodic initial conditions results to strongly chaotic motion in a short time.

From the above we see that the relative configuration between the planetary orbits, alignment or antialignment, plays a crucial role on the stability. It is the antialignment configuration that stabilizes the system, while alignment results to instability. Under the condition that the values m_1 and m_2 are small, the mass ratio m_1/m_2 does not affect the stability. The symmetry stabilizes the system and a deviation from symmetry may destabilize the system, if the deviation is large enough (see figures 11-17). Two planets in the same orbit at opposite phase form a highly unstable system and close encounters between the two planets take place during the evolution of the system.

As we found in section 2, (Fig. 4) and section 3 (Fig. 20), there is a continuous transition from satellite orbits to planetary orbits along the stable family. This means that if there is a migration mechanism in the system, due to a dissipation, it could be possible to start from satellite motion and end to planetary motion, or vice versa, since it is along a stable family that the system migrates (see Ferraz-Mello et al., 2003; Beaugé et al., 2006). We mention this just as a possible evolution. Detailed work is needed along these lines to verify whether such an evolution is indeed possible.

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