ΟΛΟΚΛΗΡΩΜΑΤΑ $\boxed{I = \int R(\cosh x,\, \sinh x)\, dx}$

$$\cosh x = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}, \qquad \sinh x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$$

$$I = \int R\left(\frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2},\, \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}\right) \mathrm{e}^{-x}\, d\left(\mathrm{e}^x\right)$$

$$\boxed{t = \mathrm{e}^x} \rightsquigarrow \boxed{\int \underbrace{R\left(\frac{t + \frac{1}{t}}{2},\, \frac{t - \frac{1}{t}}{2}\right) \frac{1}{t}}_{\text{συνάρτηση του } t}\, dt}$$

ΟΛΟΚΛΗΡΩΜΑΤΑ $\boxed{\int R(\cos x,\, \sin x)\, dx}$

$$\boxed{t = \tan \frac{x}{2}}$$

$$\cos x = 2\cos^2 \frac{x}{2} - 1 \rightsquigarrow \boxed{\cos x = \frac{1 - t^2}{1 + t^2}}$$

$$\frac{1}{\cos^2 \frac{x}{2}} = 1 + \tan^2 \frac{x}{2} = 1 + t^2$$

$$\begin{aligned} \sin x &= 2\cos \tfrac{x}{2} \sin \tfrac{x}{2} \\ &= 2\cos^2 \tfrac{x}{2} \tan \tfrac{x}{2} \end{aligned} \rightsquigarrow \boxed{\sin x = \frac{2t}{1 + t^2}}$$

$$dt = \frac{dx}{2\cos^2 \frac{x}{2}} \rightsquigarrow \boxed{dx = \frac{2dt}{1 + t^2}}$$

$$\boxed{I = \int R(\cos x,\, \sin x)\, dx = 2 \int \frac{R\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)}{1 + t^2}\, dt}$$

Ολοκλήρωμα $\boxed{\int R\left(x, \sqrt[n]{\dfrac{\alpha x+\beta}{\gamma x+\delta}}\right) dx}$

$$t^n = \frac{\alpha x+\beta}{\gamma x+\delta}$$

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Ολοκλήρωμα $\boxed{\int R\left(x, \sqrt[n]{\dfrac{\alpha x+\beta}{\gamma x+\delta}}, \sqrt[m]{\dfrac{\alpha x+\beta}{\gamma x+\delta}}\right) dx}$

$$t^p = \frac{\alpha x+\beta}{\gamma x+\delta}, \quad p = \text{Ε. Κ. Π. } (n, m)$$

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Ολοκλήρωμα $\boxed{\int R\left(x, \sqrt{a^2-x^2}\right) dx}$

$$\boxed{x = a \sin \theta}$$

$$\rightsquigarrow \; a \int R\left(a \sin \theta,\, a \cos \theta\right) \cos \theta \, d\theta$$

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Ολοκλήρωμα $\boxed{\int R\left(x, \sqrt{x^2-a^2}\right) dx}$

$$\boxed{x = a \cosh u}$$

$$\rightsquigarrow \; a \int R\left(a \cosh u,\, a \sinh u\right) \sinh u \, du$$

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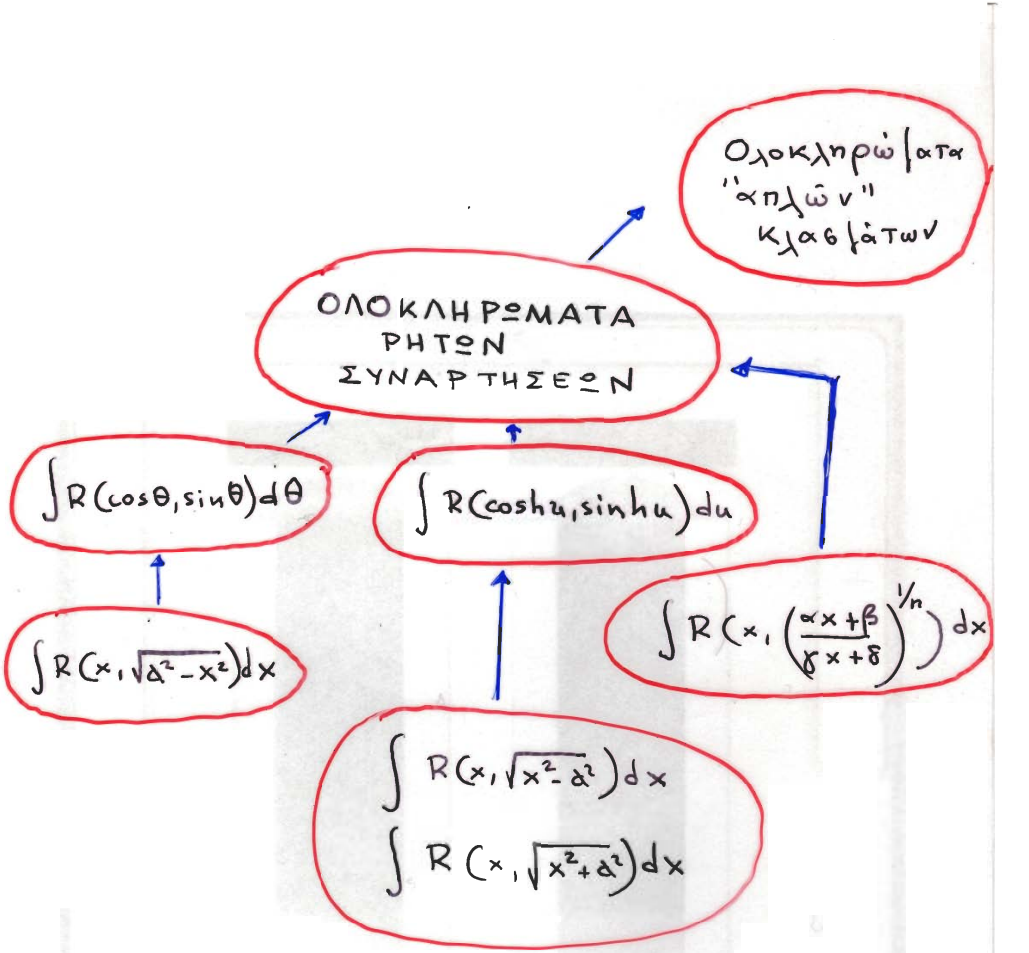
Ολοκλήρωμα $\boxed{\int R\left(x, \sqrt{a^2+x^2}\right) dx}$

$$\boxed{x = a \sinh u} \quad \text{ή} \quad x = a \tan \theta$$

$$\rightsquigarrow \; a \int R\left(a \sinh u,\, a \cosh u\right) \cosh u \, du$$


ΟΛΟΚΛΗΡΩΜΑΤΑ ΡΗΤΩΝ ΣΥΝΑΡΤΗΣΕΩΝ
Ολοκληρώματα "απλών" κλασμάτων
$\int R(\cos\theta, \sin\theta)\,d\theta$
$\int R(\cosh u, \sinh u)\,du$
$\int R\left(x, \left(\frac{\alpha x+\beta}{\gamma x+\delta}\right)^{1/n}\right)dx$
$\int R(x, \sqrt{\alpha^2 - x^2})\,dx$
$\int R(x, \sqrt{x^2 - \alpha^2})\,dx$
$\int R(x, \sqrt{x^2 + \alpha^2})\,dx$