Βασικές ιδιότητες ολοκληρωμάτων

$$\int \alpha f(x)\,dx = \alpha \int f(x)\,dx$$

Απόδειξη:

$$\frac{d\left(\int \alpha f(x)\,dx\right)}{dx} = \alpha f(x)$$

$$\Updownarrow$$

$$\frac{d\left(\alpha \int f(x)\,dx\right)}{dx} = \alpha \frac{d\left(\int f(x)\,dx\right)}{dx} = \alpha f(x)$$

$$\int \{f(x) + g(x)\}\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Απόδειξη:

$$\frac{d\{\int (f(x)+g(x))\,dx\}}{dx} = f(x) + g(x)$$

$$\Updownarrow$$

$$\frac{d\{\int f(x)\,dx + \int g(x)\,dx\}}{dx} =$$

$$= \frac{d\left(\int f(x)\,dx\right)}{dx} + \frac{d\left(\int g(x)\,dx\right)}{dx} = f(x) + g(x)$$

Ολοκλήρωση με αντικατάσταση μεταβλητής:

$$\int g(u)\,du \underset{u=u(x)}{=\joinrel=} \int g\left(u(x)\right)u'(x)\,dx =$$
$$= \int g\left(u(x)\right)\frac{du(x)}{dx}\,dx =$$
$$= \int g\left(u(x)\right)\,du(x)$$

πχ

$$\int \tan x\,dx = \int \frac{\sin x\,dx}{\cos x} = -\int \frac{d\left(\cos x\right)}{\cos x} =$$

$$\underset{u=\cos x}{=\joinrel=} -\int \frac{du}{u} = -\ln|u| + c =$$

$$= \ln \frac{1}{|\cos x|} + c$$

$$\int \frac{x}{\sqrt{x^2 + a^2}}\,dx = \frac{1}{2}\int \frac{d\left(x^2 + a^2\right)}{\sqrt{x^2 + a^2}} =$$

$$\underset{u=x^2+a^2}{=\joinrel=} \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} + c =$$
$$= \sqrt{x^2 + a^2} + c$$

Ολοκλήρωση κατά παράγοντες:

$$\int d(fg) = \int fdg + \int gdf$$

$$\rightsquigarrow \boxed{\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int g(x)\,f'(x)\,dx}$$

πχ

$$\int x\,\mathrm{e}^{-x}\,dx = -\int x\,d\left(\mathrm{e}^{-x}\right) =$$
$$= -x\,\mathrm{e}^{-x} + \int \mathrm{e}^{-x}\,dx = -x\,\mathrm{e}^{-x} - \mathrm{e}^{-x}$$

$$\int \ln x\,dx = x\,\ln x - \int x\,d\left(\ln x\right) =$$
$$= x\,\ln x - \int \frac{x}{x}\,dx = x\,\ln x - x$$