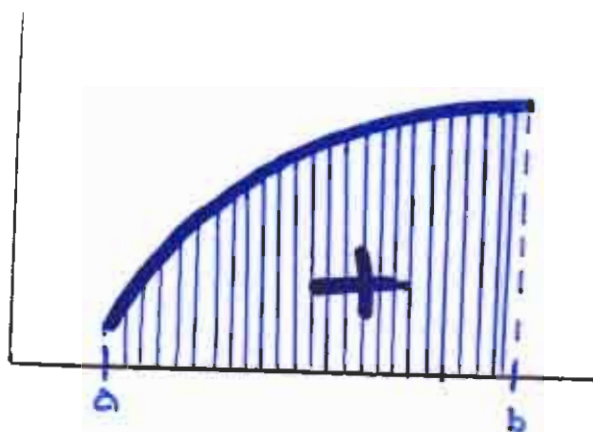


Εφ. ΕΜΒΑΔΟΝ ΜΕΤΑΞΥ ΚΑΜΠΥΛΩΝ

$$f(x) \geq 0$$

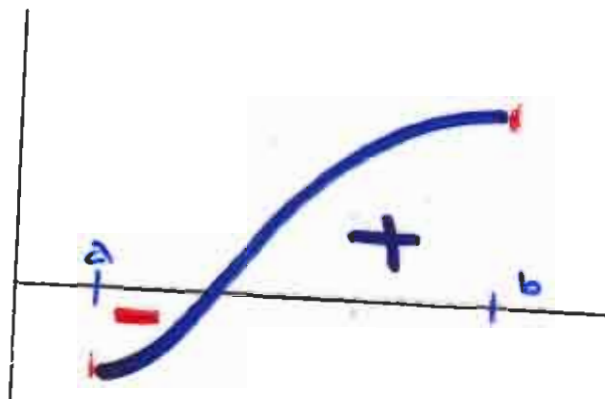
$$\text{Εμβαδόν} = \int_a^b f(t) dt$$



Προσημασμένο

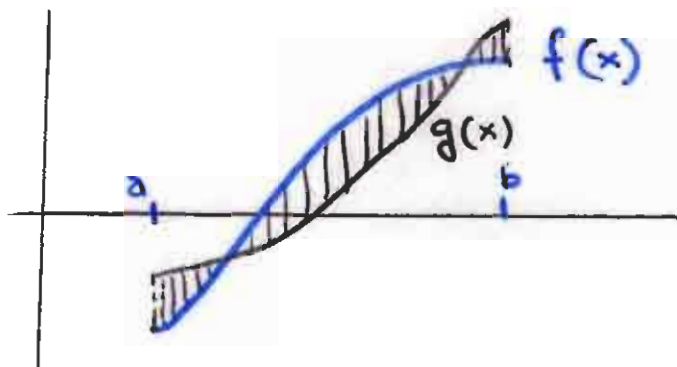
Εμβαδόν

$$\int_a^b f(x) dx$$



Εμβαδόν μεταξύ
δύο καμπύλων

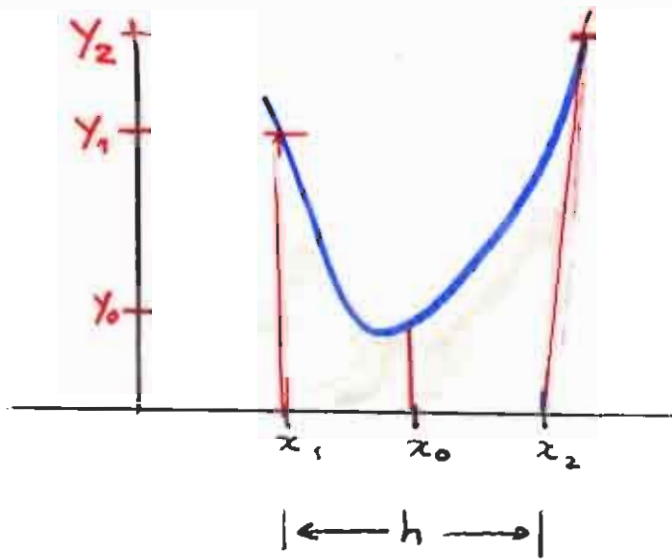
$$E = \int_a^b |f(x) - g(x)| dx$$



$$M(x) = \max\{f(x), g(x)\} = \frac{1}{2} (|f(x) - g(x)| + f(x) + g(x))$$

$$m(x) = \min\{f(x), g(x)\} = \frac{1}{2} (-|f(x) - g(x)| + f(x) + g(x))$$

$$E = \int_a^b (M(x) - m(x)) dx = \int_a^b |f(x) - g(x)| dx$$

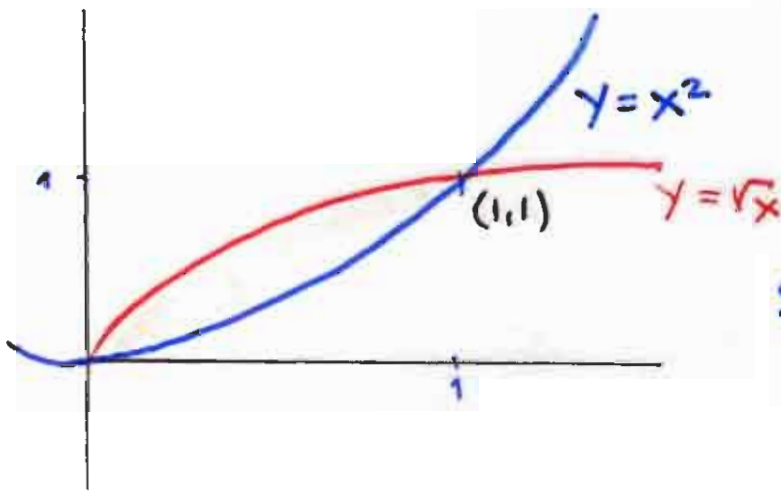


$$y = x^2 + \alpha x + \beta$$

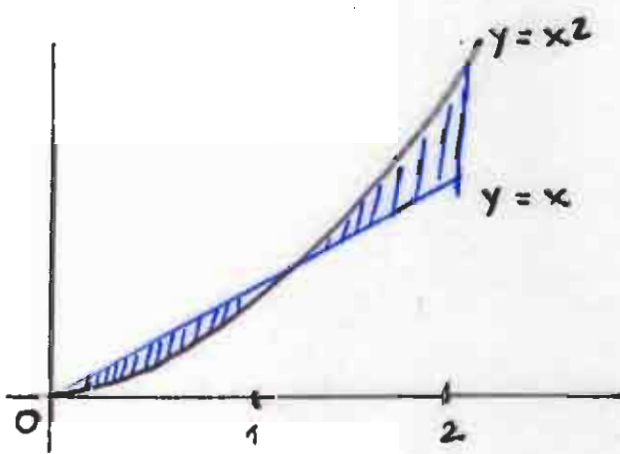
$$\Delta = \alpha^2 - 4\beta < 0$$

$$S = \int_{x_1}^{x_2} y dx = \frac{h}{6} (y_1 + y_0 + 4y_2)$$

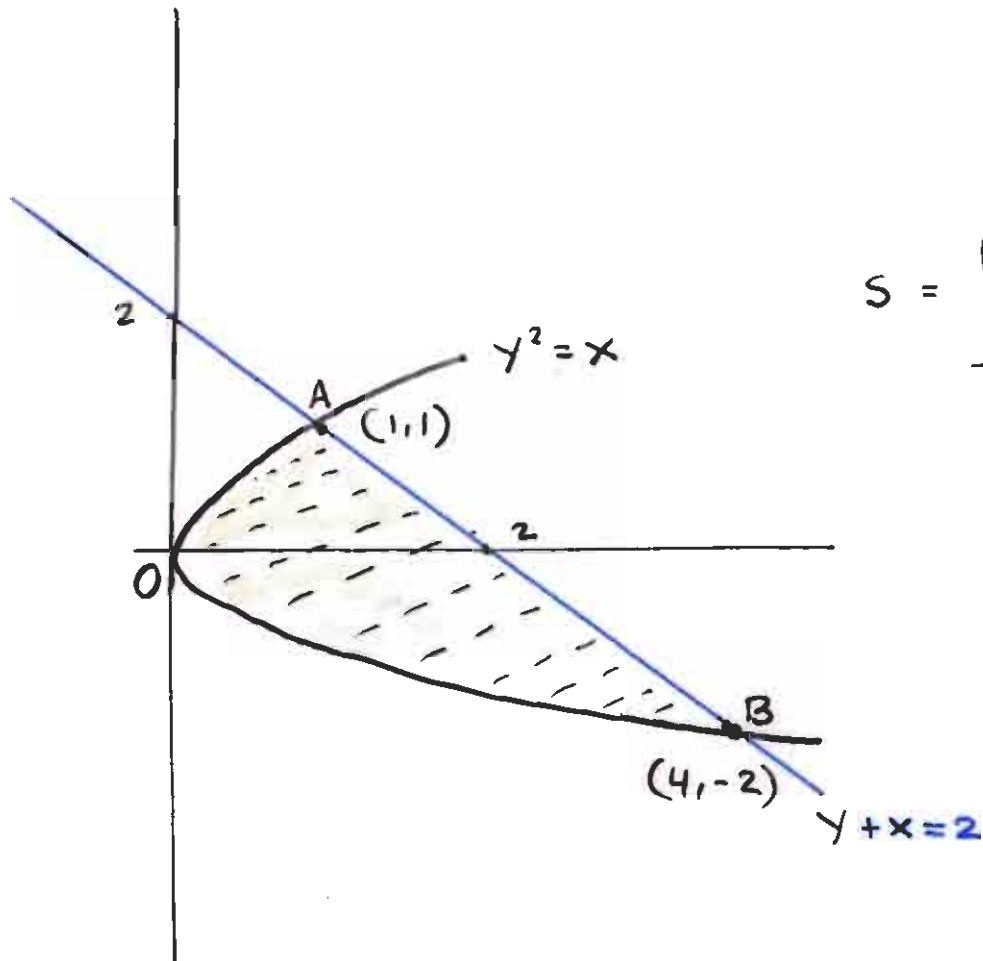
Επιβάθμιση



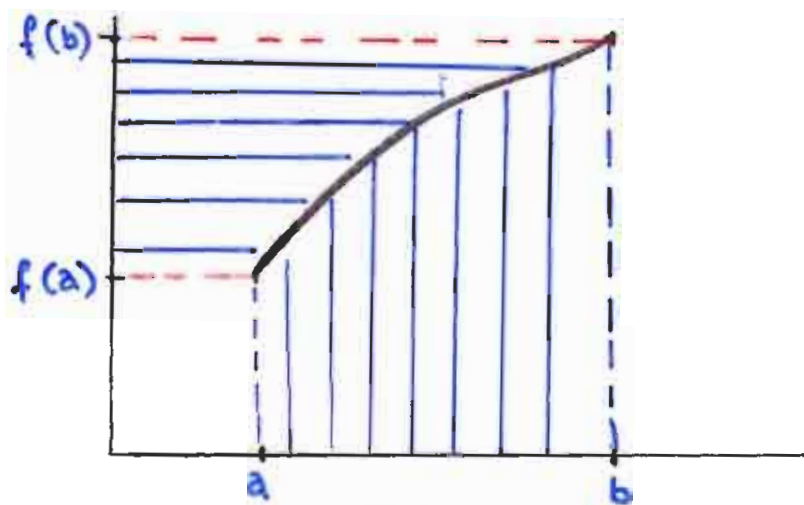
$$S = \int_0^1 (\sqrt{x} - x^2) dx = \frac{1}{6}$$



$$S = \int_0^2 |x^2 - x| dx$$



$$S = \int_{-2}^1 |y^2 - (2-y)| dy$$



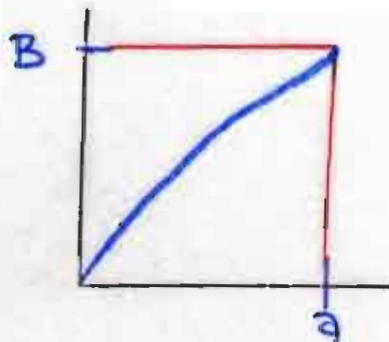
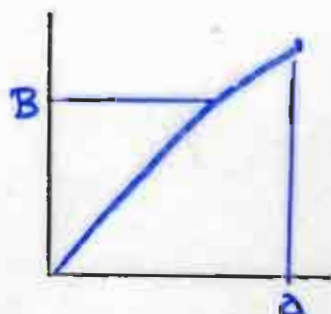
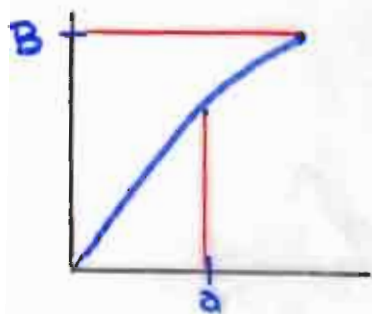
$$f'(x) > 0$$

$$\int_a^b f(x) dx + \int_{f(a)}^{f(b)} f^{-1}(u) du = bf(b) - af(a)$$

"Ανισότητα Young"

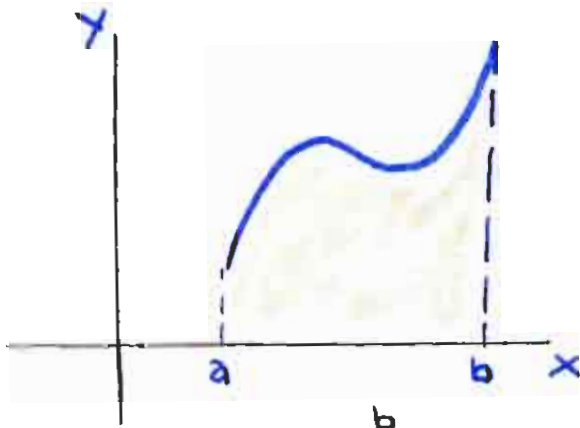
$$f'(x) > 0 \quad \Delta \in [0, c]$$

$$f(0) = 0 \quad B \in [0, f(c)]$$



$$aB \leq \int_0^a f(x) dx + \int_0^B f^{-1}(u) du$$

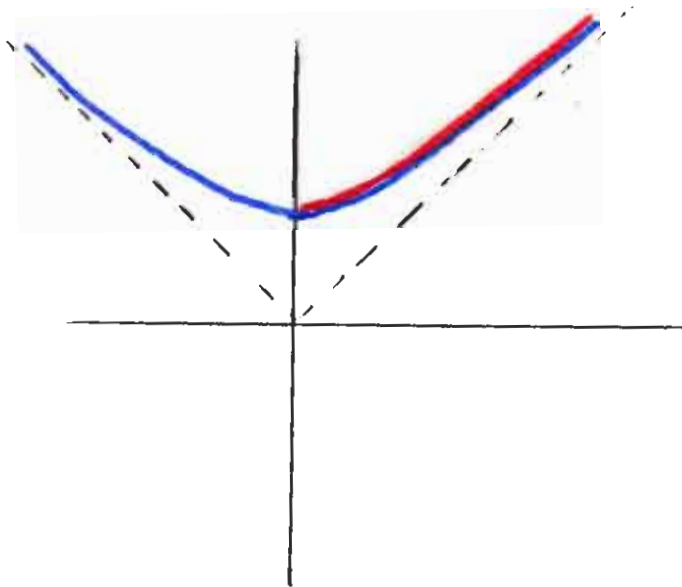
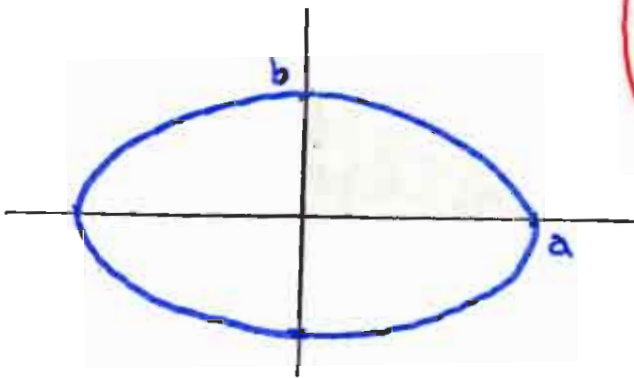
ΕΜΒΑΔΟΝ ΚΑΜΠΥΛΗΣ ΣΕ ΠΑΡΑΜΕΤΡΙΚΗ ΜΟΡΦΗ



$$\begin{aligned}x &= x(t) & t &\in [\alpha, \beta] \\y &= y(t) & x(\alpha) &= a \\ & & x(\beta) &= b\end{aligned}$$

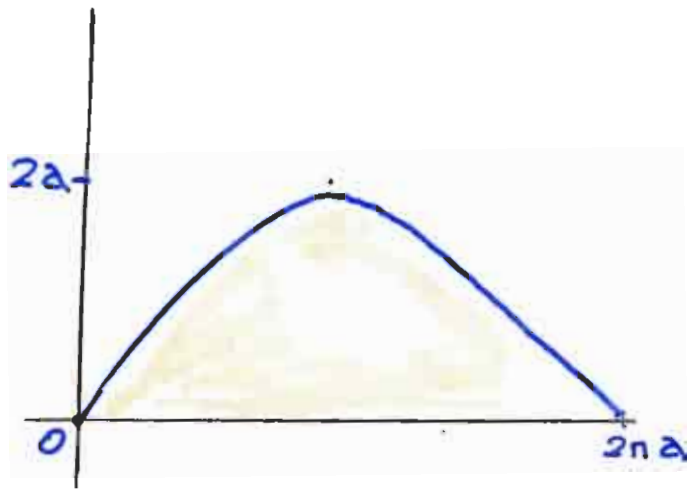
$$S = \int_a^b y \, dx = \int_{\alpha}^{\beta} y(t) x'(t) \, dt$$

ΕΛΛΕΙΨΗ $(x/a)^2 + (y/b)^2 = 1$
 $x = a \cos t$
 $y = b \sin t$



ΥΠΕΡΒΟΛΗ $(y/a)^2 - (x/b)^2 = 1$

$$\begin{aligned}y &= a \cosh t \\x &= b \sinh t\end{aligned}$$



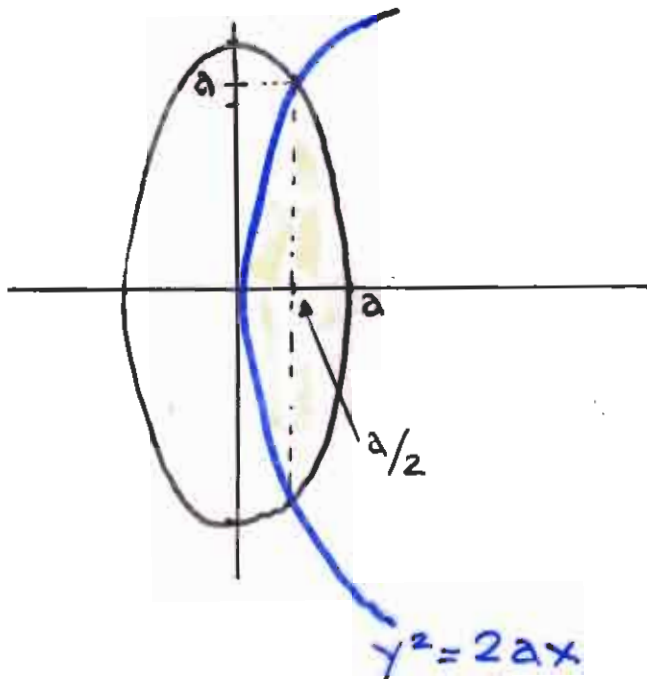
ΕΜΒΑΔΟΝ
ΚΥΚΛΟΕΙΔΟΥΣ

$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$

$$0 \leq t \leq 2\pi$$

$$S = 3\pi a^2$$



$$4x^2 + 3y^2 = 4a^2$$

$$y^2 = 2ax$$

Εξίσωση ελλείψης

$$x = a \cos t$$

$$y = \frac{2}{\sqrt{3}} a \sin t$$

$$S = 2a^2 \left(\frac{1}{12} + \frac{\pi}{3\sqrt{3}} \right)$$